SET 1

(1) Solve for x:

(a) $e^{2^x} = 5^{3^x}$

Solution:
We take natural log on both sides:

$$\ln(e^{2^x}) = \ln(5^{3^x})$$
$$2^x = 3^x \ln(5)$$

Now we take log base 2 on both sides:

$$\log_2(2^x) = \log_2(3^x \ln 5)$$
$$x = \log_2(3^x) + \log_2(\ln(5))$$
$$x - x\log_2 3 = \log_2(\ln 5)$$
$$x(1 - \log_2 3) = \log_2(\ln 5)$$
$$x = \frac{\log_2(\ln 5)}{1 - \log_2 3}$$

Since the initial equation is defined on all real numbers, plugging this solution in the initial equation makes sense, so this is indeed a solution.

(b) $\log_2(x+1) + \log_2(x+2) = 1$

Solution:
This equation is defined when the arguments inside the logarithms are positive, that is, when $x+1 > 0$ and $x+2 > 0$. Thus, the solutions will be in the interval $(-1, +\infty)$. We proceed to solve the equation:

$$\log_2(x+1)(x+2) = 1$$

We raise 2 to the power of both sides

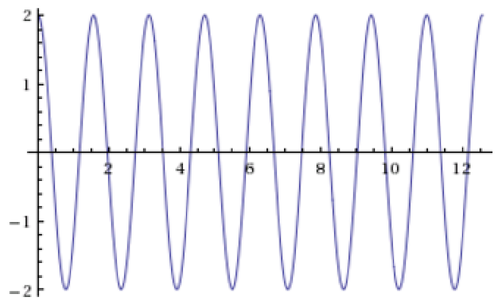
$$2^{\log_2(x+1)(x+2)} = 2^1$$
$$(x+1)(x+2) = 2$$
$$x^2 + 3x + 2 = 2$$
$$x^2 + 3x = 0$$
$$x(x+3) = 0$$

So the possible solutions are $x = 0$ and $x = -3$. But since we were looking only at solutions in the interval $(-1, +\infty)$, the only solution is $x = 0$.

(2) Let $f(x) = 2\cos(4x)$. What is the amplitude? What is the period? Graph the function on the interval $[0, 4\pi]$

Solution:

The amplitude is 2, the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. In the graph below, note that since the period is $\pi/2$, we have the portion of the graph from 0 to $\pi/2$ ($\pi/2$ is approximately 1.6) repeating itself 8 times in the interval $[0, 4\pi]$



(3) Solve for x:

$$\tan\left(x + \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$$

Solution:

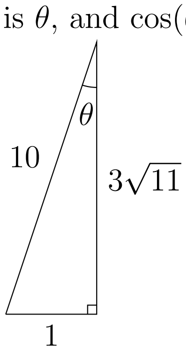
We know that the angle $\pi/6$ has tangent $\frac{1}{\sqrt{3}}$, and that we obtain all such angles by going around the circle additional multiples of π in both directions. Therefore,

$$x + \frac{\pi}{2} = \frac{\pi}{6} + k\pi, \qquad k \in \mathbb{Z}$$
$$x = \frac{\pi}{6} - \frac{\pi}{2} + k\pi, \qquad k \in \mathbb{Z}$$
$$x = \frac{\pi - 3\pi}{6} + k\pi, \qquad k \in \mathbb{Z}$$
$$x = \frac{-\pi}{3} + k\pi, \qquad k \in \mathbb{Z}$$

(4) Draw the appropriate right triangle and evaluate

$$\cos\left(\arcsin\left(\frac{1}{10}\right)\right)$$

Solution: We get the following triangle, where the third side length is given by the Pythagorean Theorem : $\sqrt{10^2 - 1^2} = \sqrt{99} = 3\sqrt{11}$. The angle we are interested in is θ, and $\cos(\theta) = \frac{3\sqrt{11}}{10}$.



SET 2

(1) Solve for x:

(a) $2^{3x+3} = 5^{2x+1}$

Solution:
We take log base 2 on both sides:

$$\log_2(2^{3x+3}) = \log_2(5^{2x+1})$$
$$3x + 3 = (2x + 1)\log_2 5$$
$$3x - 2x\log_2 5 = \log_2 5 - 3$$
$$x(3 - 2\log_2 5) = \log_2 5 - 3$$
$$x = \frac{\log_2 5 - 3}{3 - 2\log_2 5}$$

Since the initial equation is defined on all real numbers, plugging this solution in the initial equation makes sense, so this is indeed a solution.

(b) $\ln(x + 7) - \ln(x + 1) = \ln(x - 3)$

Solution:
This equation is defined when the arguments inside the logarithms are positive, that is, when $x + 7 > 0$, $x + 1 > 0$ and $x - 3 > 0$. Thus, the solutions will be in the interval $(3, +\infty)$. We proceed to solve the equation:

$$\ln\frac{x + 7}{x + 1} = \ln(x - 3)$$

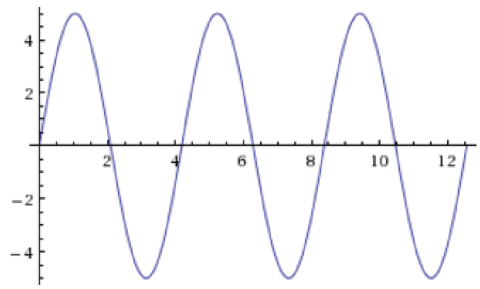
We raise e to the power of both sides

$$e^{\ln\frac{x+7}{x+1}} = e^{\ln(x-3)}$$
$$\frac{x + 7}{x + 1} = x - 3$$
$$x + 7 = (x - 3)(x + 1)$$
$$x + 7 = x^2 - 2x - 3$$
$$x^2 - 3x - 10 = 0$$

Using the quadratic formula, we have the possible solutions $x = -2$ and $x = 5$. But since we were looking only at solutions in the interval $(3, +\infty)$, the only solution is $x = 5$.

(2) Let $f(x) = 5\sin\left(\frac{3x}{2}\right)$. What is the amplitude? What is the period? Graph the function on the interval $[0, 4\pi]$

Solution: The amplitude is 5, the period is $\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$. In the graph below, note that since the period is $4\pi/3$, we have the portion of the graph from 0 to $4\pi/3$ ($4\pi/3$ is approximately 4.2) repeating itself 3 times in the interval $[0, 4\pi]$



(3) Solve for x:

$$\cos(x) = \frac{1}{\sqrt{2}}$$

Solution:

We know two angles that have cosine $\frac{1}{\sqrt{2}}$: $\frac{\pi}{4}$ and $\frac{-\pi}{4}$. We obtain half the angles that have cosine $\frac{1}{\sqrt{2}}$ by going around the circle multiples of 2π from $\pi/4$, and we obtain the other half of angles by going around the circle additional multiples of 2π from $-\pi/4$. Therefore, we have two kinds of angles:

$$\frac{\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$$

and

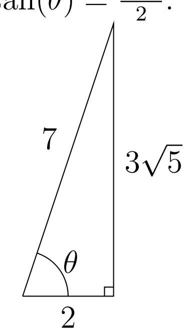
$$\frac{-\pi}{4} + 2k\pi, \quad k \in \mathbb{Z}$$

(4) Draw the appropriate right triangle and evaluate

$$\tan\left(\arccos\left(\frac{2}{7}\right)\right)$$

Solution:

We get the following triangle, where the third side length is given by the Pythagorean Theorem : $\sqrt{7^2 - 2^2} = \sqrt{45} = 3\sqrt{5}$. The angle we are interested in is θ, and $\tan(\theta) = \frac{3\sqrt{5}}{2}$.



SET 3

(1) Solve for x:

(a) $2^{3^x} = 4^{2^x}$

Solution: We rewrite $2^{3^x} = 4^{2^x}$ as $2^{3^x} = (2^2)^{2^x}$, so $2^{3^x} = 2^{2 \cdot 2^x}$. Now we take log base 2 from both sides, and

$$\log_2(2^{3^x}) = \log_2(2^{2 \cdot 2^x})$$
$$3^x = 2 \cdot 2^x$$
$$3^x = 2^{x+1}$$

Now we take log base 2 again on both sides, and we get

$$\log_2(3^x) = \log_2(2^{x+1})$$
$$x \log_2 3 = x + 1$$
$$x \log_2 3 - x = 1$$
$$x(\log_2 3 - 1) = 1$$
$$x = \frac{1}{\log_2 3 - 1}$$

Since the initial equation is defined on all real numbers, plugging this solution in the initial equation makes sense, so this is indeed a solution.

(b) $\log_{10}(x + 3) + \log_{10}(x + 4) = \log_{10}(6)$

Solution:
This equation is defined when the arguments inside the logarithms are positive, that is, when $x+3 > 0$ and $x+4 > 0$. Thus, the solutions will be in the interval $(-3, +\infty)$. We proceed to solve the equation:

$$\log_{10}(x + 3)(x + 4) = \log_{10}(6)$$

We raise 10 to the power of both sides

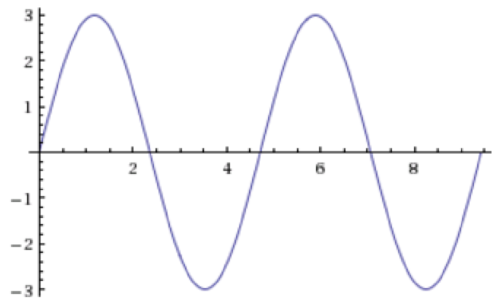
$$10^{\log_{10}(x+3)(x+4)} = 10^{\log_{10}(6)}$$
$$(x + 3)(x + 4) = 6$$
$$x^2 + 7x + 12 = 6$$
$$x^2 + 7x + 6 = 0$$

Using the quadratic formula, we get the possible solutions to be $x = -1$ and $x = -6$. But since we were looking only at solutions in the interval $(-3, +\infty)$, the only solution is $x = -1$.

(2) Let $f(x) = 3\sin\left(\frac{4x}{3}\right)$. What is the amplitude? What is the period? Graph the function on the interval $[0, 3\pi]$

Solution:

The amplitude is 3, the period is $\frac{2\pi}{\frac{4}{3}} = \frac{3\pi}{2}$. In the graph below, note that since the period is $3\pi/2$, we have the portion of the graph from 0 to $3\pi/2$ ($3\pi/2$ is approximately 4.7) repeating itself 2 times in the interval $[0, 4\pi]$



(3) Solve for x:

$$\sin(x) = -\frac{1}{2}$$

Solution:

We know two angles that have sine $-\frac{1}{2}$: $-\frac{\pi}{6}$ and $\pi + \frac{\pi}{6}$. We obtain half the angles that have sine $-\frac{1}{2}$ by going around the circle multiples of 2π from $-\frac{\pi}{6}$, and we obtain the other half of angles by going around the circle additional multiples of 2π from $\pi + \frac{\pi}{6}$. Therefore, we have two kinds of angles:

$$-\frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

and

$$\pi + \frac{\pi}{6} + 2k\pi, \quad k \in \mathbb{Z}$$

(4) Draw the appropriate right triangle and evaluate

$$\sin(\arctan(5))$$

Solution:

We get the following triangle, where the third side length is given by the Pythagorean Theorem : $\sqrt{1^2 + 5^2} = \sqrt{26}$. The angle we are interested in is θ, and $\sin(\theta) = \frac{5}{\sqrt{26}}$.