DIFFERENTIATION WORKSHEET

Exercise 1. Calculate the derivative using the limit definition for the following functions:

(1) $f(x) = x + 3$

Solution:

$$\frac{d}{dx}(x+3) = \lim_{h\to 0} \frac{(x+h+3)-(x+3)}{h} = \lim_{h\to 0} \frac{x+h+3-x-3}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1$$

(2) $g(x) = 2x - 5$

Solution:

$$\frac{d}{dx}(2x^2+x-15) = \lim_{h\to 0} \frac{(2(x+h)-5)-(2x-5)}{h} = \lim_{h\to 0} \frac{2x+2h-5-2x+5}{h} = \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = 2$$

(3) $h(x) = 2x^2 + x - 15$. Check that $h(x) = f(x)g(x)$, and that $h'(x) = f'(x)g(x) + f(x)g'(x)$

Solution:

$$\frac{d}{dx}(2x-5) = \lim_{h\to 0} \frac{2(x+h)^2+(x+h)-15)-(2x^2+x-15)}{h} =$$

$$\lim_{h\to 0} \frac{2x^2+4hx+2h^2+x+h-15-2x^2-x+15}{h} = \lim_{h\to 0} \frac{4hx+2h^2+h}{h} = \lim_{h\to 0}(4x+2h+1) = 4x+1$$

Then $f'(x)g(x)+f(x)g'(x) = 1(2x-5)+(x+3)\cdot 2 = 2x-5+2x+6 = 4x+1 = h'(x)$.

(4) **Challenge:** $j(x) = \frac{1}{x+1}$

Solution:

$$\frac{d}{dx}\left(\frac{1}{x+1}\right) = \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{x+1-(x+h+1)}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0} \frac{x+1-x-h-1}{(x+h+1)(x+1)h} =$$

$$\lim_{h\to 0} \frac{-h}{(x+h+1)(x+1)} = \frac{-1}{(x+1)(x+1)} = \frac{-1}{(x+1)^2}$$

Exercise 2. Calculate the derivative using the differentiation rules for the following functions:

(1) $f(x) = 15x^{20} - 5x^{10} + 6x$

Solution: Using the sum/difference rules, we can separate the terms in the functions, and have

$$\frac{d}{dx}(15x^{20} - 5x^{10} + 6x) = \frac{d}{dx}(15x^{20}) - \frac{d}{dx}(5x^{10}) + \frac{d}{dx}(6x)$$

Now, we can use the Constant Multiple rule,

$$\frac{d}{dx}(15x^{20}) - \frac{d}{dx}(5x^{10}) + \frac{d}{dx}(6x) = 15\frac{d}{dx}(x^{20}) - 5\frac{d}{dx}(x^{10}) + 6\frac{d}{dx}(x)$$

By the Power rule, this equality extends to

$$15\frac{d}{dx}(x^{20}) - 5\frac{d}{dx}(x^{10}) + 6\frac{d}{dx}(x) = 15(20x^{19}) - 5(10x^9) + 6$$

Therefore, $f'(x) = 15(20x^{19}) - 5(10x^9) + 6$

(2) $g(x) = 5e^x + x^3 - 4x^2$

Solution: Using the sum/difference rules, we can separate the terms in the functions, and have

$$\frac{d}{dx}(5e^x + x^3 - 4x^2) = \frac{d}{dx}(5e^x) + \frac{d}{dx}(x^3) - \frac{d}{dx}(4x^2)$$

Now, we can use the Constant Multiple rule,

$$\frac{d}{dx}(5e^x) + \frac{d}{dx}(x^3) - \frac{d}{dx}(4x^2) = 5\frac{d}{dx}(e^x) + \frac{d}{dx}(x^3) - 4\frac{d}{dx}(x^2)$$

By the Power rule and the formula for the derivative of the exponential, this equality extends to

$$5\frac{d}{dx}(e^x) + \frac{d}{dx}(x^3) - 4\frac{d}{dx}(x^2) = 5e^x + 3x^2 - 4(2x)$$

and $g'(x) = 5e^x + 3x^2 - 8x$

(3) $h(x) = 2x^2 + x - 15$

Solution: Using the sum/difference and constant multiple rules,

$$h'(x) = 2(2x) + 1 = 4x + 1,$$

the same as we got in Exercise 1. part (3)!

(4) $j(x) = 2x^{-5} + \frac{1}{x} + 6e^x$

Solution: Using the sum/difference and constant multiple rules, as well as the formula for the derivative of the exponential,

$$j'(x) = 2(-5)x^{-6} + (-1)x^{-2} + 6e^x = -10x^{-6} - x^{-2} + 6e^x$$