Implicit Differentiation Practice

1. Find the equation of the tangent line to the curve

$$x^2 + xy + y^2 = 3.$$

at $(1, 1)$.

Answer:

We first take the derivative of both sides. Doing so gives us,

$$2x + \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0.$$

Solving for dy/dx gives us

$$\frac{dy}{dx} = -\frac{2x + y}{2y + x}$$

Plugging in the point $(1, 1)$ gives

$$\frac{dy}{dx} = -\frac{2(1) + 1}{2(1) + 1} = -1.$$

Thus the equation of the tangent line to the curve at $(1, 1)$ is

$$t(x) = -1(x - 1) + 1 = -x + 2$$

2. Find the equation of the tangent line to the curve

$$x^2 + 2xy - y^2 + x = 2.$$

at $(1, 2)$.

Answer:

We first take the derivative of both sides. Doing so gives us,

$$2x + 2\left(x\frac{dy}{dx} + y\right) - 2y\frac{dy}{dx} + 1 = 0.$$

Solving for dy/dx gives us

$$\frac{dy}{dx} = -\frac{2x + 2y + 1}{2x - 2y}$$

Plugging in the point $(1, 2)$ gives

$$\frac{dy}{dx} = -\frac{2(1) + 2(2) + 1}{2(1) - 2(2)} = \frac{7}{2}.$$

Thus the equation of the tangent line to the curve at $(1, 1)$ is

$$t(x) = \frac{7}{2}(x - 1) + 2 = \frac{7}{2}x - \frac{3}{2}.$$

3. Find the equation of the tangent line to the curve

$$x^{2/3} + y^{2/3} = 4.$$

at $(-3\sqrt{3}, 1)$.

Answer:

We first take the derivative of both sides. Doing so gives us,

$$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = 0.$$

Solving for dy/dx gives us

$$\frac{dy}{dx} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

Plugging in the point $(-3\sqrt{3}, 1)$ gives

$$\frac{dy}{dx} = -\frac{\sqrt[3]{1}}{\sqrt[3]{-3\sqrt{3}}} = \frac{1}{-\sqrt[3]{3^{3/2}}} = -\frac{1}{\sqrt{3}}.$$

Thus the equation of the tangent line to the curve at $(1, 1)$ is

$$t(x) = -\frac{1}{\sqrt{3}}(x - (-3\sqrt{3})) + 1 = -\frac{1}{\sqrt{3}}x - 2$$