Trigonometric Derivative Practice

Compute the following derivatives.

1. Find the derivative of
$$f(x) = \cot(x).$$

   Answer:

   We know $\cot(x) = \frac{\cos(x)}{\sin(x)}$. Thus by the quotient rule we have

$$\begin{aligned} g'(x) &= \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{\sin(x)\frac{d}{dx}(\cos(x)) - \cos(x)\frac{d}{dx}(\sin(x))}{(\sin(x))^2} \\ &= \frac{-(\sin(x))^2 - (\cos(x))^2}{(\sin(x))^2} \\ &= \frac{-1}{(\sin(x))^2} \\ &= -(\csc(x))^2. \end{aligned}$$

2. Find the derivative of
$$g(x) = (\cos(x))^2.$$

   Answer:

   We may think of $(\cos(x))^2$ as $\cos(x)\cos(x)$ and use the product rule.

$$\begin{aligned} g'(x) &= \frac{d}{dx}(\cos(x)\cos(x)) \\ &= \cos(x)\frac{d}{dx}(\cos(x)) + \cos(x)\frac{d}{dx}(\cos(x)) \\ &= \cos(x)(-sin(x)) + \cos(x)(-sin(x)) \\ &= -2\cos(x)\sin(x). \end{aligned}$$

3. Find the derivative of

$$\ell(x) = \cos(x)x^2e^x.$$

Answer:

We know that

$$\frac{d}{dx}\cos(x) = -\sin(x), \qquad \frac{d}{dx}\left(x^2\right) = 2x, \qquad \text{and} \qquad \frac{d}{dx}e^x = e^x$$

Thus

$$\frac{d}{dx}(\cos(x)x^2) = \cos(x)\frac{d}{dx}(x^2) + x^2\frac{d}{dx}(\cos(x)) = 2x\cos(x) - x^2\sin(x).$$

Thus,

$$\begin{aligned}\ell'(x) = \frac{d}{dx}(\cos(x)x^2e^x) &= \frac{d}{dx}\left(\left(\cos(x)x^2\right)e^x\right)\\ &= \cos(x)x^2\frac{d}{dx}\left(e^x\right) + e^x\frac{d}{dx}\left(\cos(x)x^2\right)\\ &= \cos(x)x^2e^x + e^x\left(2x\cos(x) - x^2\sin(x)\right)\\ &= \cos(x)x^2e^x + \cos(x)2xe^x - \sin(x)x^2e^x\end{aligned}$$

4. Find the derivative of

$$k(x) = \frac{x^3}{\sin(x)e^x}.$$

Answer:

We know that

$$\frac{d}{dx}\sin(x) = \cos(x), \qquad \frac{d}{dx}\left(x^3\right) = 3x^2, \qquad \text{and} \qquad \frac{d}{dx}e^x = e^x$$

Thus

$$\frac{d}{dx}(\sin(x)e^x) = \sin(x)\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(\sin(x)) = e^x\sin(x) + e^x\cos(x).$$

Thus by the quotient rule,

$$\begin{aligned}k'(x) = \frac{d}{dx}\left(\frac{x^3}{\sin(x)e^x}\right) &= \frac{\sin(x)e^x\frac{d}{dx}(x^3) - x^3\frac{d}{dx}\left(\sin(x)e^x\right)}{\left(\sin(x)e^x\right)^2}\\ &= \frac{3x^2\sin(x)e^x - x^3\left(e^x\sin(x) + e^x\cos(x)\right)}{\left(\sin(x)e^x\right)^2}\\ &= \frac{3x^2\sin(x) - x^3\sin(x) - x^3\cos(x)}{\left(\sin(x)\right)^2e^x}.\end{aligned}$$