Derivative Laws Practice

Using the laws of derivatives, compute the following derivatives.

1. Find the derivative of

$$f(x) = x^c e^x,$$

for c a real number.

Answer:

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x^c e^x) = e^x \frac{d}{dx}(x^c) + x^c \frac{d}{dx}(e^x) \\ &= c x^{c-1} e^x + x^c e^x \end{aligned}$$

2. Find the derivative of

$$g(x) = \frac{e^x}{x^4}.$$

Answer:

$$\begin{aligned} g'(x) &= \frac{x^4 \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x^4)}{(x^4)^2} \\ &= \frac{x^4 e^x - 4x^3 e^x}{x^8} \\ &= \frac{x^3 e^x \,(x-4)}{x^8} \\ &= \frac{e^x (x-4)}{x^5} \end{aligned}$$

3. Find the second derivative of
$$\ell(x) = x^c,$$
for c a real number

Answer:

$$\begin{aligned}
\ell'(x) &= \frac{d}{dx}\left(\frac{d}{dx}\left(x^c\right)\right) \\
&= \frac{d}{dx}\left(cx^{c-1}\right) \\
&= c\frac{d}{dx}\left(x^{c-1}\right) \\
&= c(c-1)x^{c-2}
\end{aligned}$$

4. Find the derivative of
$$k(x) = a^x b^x,$$
where a, b are positive numbers.

Answer:

There are two ways of doing this:

Option 1 (product rule):

$$\begin{aligned}
k'(2) &= \frac{d}{dx}(a^x b^x) = a^x \frac{d}{dx}(b^x) + b^x \frac{d}{dx}(a^x) \\
&= \ln(b) \cdot a^x b^x + \ln(a) \cdot a^x b^x
\end{aligned}$$

Option 2 (exponent laws):

$$\begin{aligned}
k'(2) &= \frac{d}{dx}(a^x b^x) = \frac{d}{dx}((ab)^x) \\
&= \ln(ab) \cdot (ab)^x
\end{aligned}$$

Observe that

$$\ln(b) \cdot a^x b^x + \ln(a) \cdot a^x b^x = (\ln(b) + \ln(a)) \cdot a^x b^x = \ln(ab) \cdot a^x b^x = \ln(ab) \cdot (ab)^x,$$

and so the two answers are in fact equal.