

# Derivative as a Function Practice

Compute the derivative of the following functions. Show all your work.

1. Find the derivative of

$$f(x) = 3x^2 - 1.$$

**Answer:**

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 1 - (3x^2 - 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 1 - 3x^2 + 1}{h} = \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} (6x + h) = 6x \end{aligned}$$

2. Find the derivative of

$$g(x) = x^3.$$

**Answer:**

$$\begin{aligned} g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = 3x^2 \end{aligned}$$

3. Find the derivative of

$$\ell(x) = 5\sqrt{x}.$$

**Answer:**

$$\begin{aligned}\ell'(x) &= \lim_{h \rightarrow 0} \frac{\ell(5+h) - \ell(5)}{h} = \lim_{h \rightarrow 0} \frac{5\sqrt{x+h} - 5\sqrt{x}}{h} \\ &= 5 \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left( \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) = 5 \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\ &= 5 \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = 5 \lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})} \\ &= \frac{5}{\sqrt{x} + \sqrt{x}} = \frac{5}{2\sqrt{x}}\end{aligned}$$

4. Find the derivative of

$$k(x) = \frac{3}{x-2}.$$

**Answer:**

$$\begin{aligned}k'(2) &= \lim_{h \rightarrow 0} \frac{k(x+h) - k(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3}{x-2+h} - \frac{3}{x-2}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3}{x-2+h} - \frac{3}{x-2} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3(x-2)}{(x-2)(x-2+h)} - \frac{3(x-2+h)}{(x-2)(x-2+h)} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3(x-2) - 3(x-2+h)}{(x-2)(x-2+h)} \right) = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{-3h}{(x-2)(x-2+h)} \right) \\ &= \lim_{h \rightarrow 0} \frac{-3}{(x-2)(x-2+h)} = \frac{-3}{(x-2)^2}\end{aligned}$$