Derivative at a Point Practice

Compute the derativative of the following functions at the given point. Show all your work.

1. Find the derivative of

$$f(x) = x^2 + 2$$

at 4.

Answer:

$$f'(4) = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{(4+h)^2 + 2 - (4^2+2)}{h} = \lim_{h\to 0} \frac{4^2 + 2(4)h + h^2 - 4^2}{h}$$
$$= \lim_{h\to 0} \frac{8h - h^2}{h} = \lim_{h\to 0} (8 - h) = 8$$

2. Find the derivative of

$$3x^2$$

at 1

Answer:

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{3(1+h)^2 - 3(1)^2}{h} = \lim_{h\to 0} \frac{3(1)^2 + 6(1)h + 3h^2 - 3}{h}$$
$$= \lim_{h\to 0} \frac{6h - h^2}{h} = \lim_{h\to 0} (6 - h) = 6$$

3. Find the derivative of

$$\sqrt{x}$$

at 5

Answer:

$$\begin{aligned}
f'(5) &= \lim_{h\to 0}\frac{f(5+h)-f(5)}{h} = \lim_{h\to 0}\frac{\sqrt{5+h}-\sqrt{5}}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{5+h}-\sqrt{5}}{h}\left(\frac{\sqrt{5+h}+\sqrt{5}}{\sqrt{5+h}+\sqrt{5}}\right) = \lim_{h\to 0}\frac{(5+h)-5}{h\left(\sqrt{5+h}+\sqrt{5}\right)} \\
&= \lim_{h\to 0}\frac{h}{h\left(\sqrt{5+h}+\sqrt{5}\right)} = \lim_{h\to 0}\frac{1}{\left(\sqrt{5+h}+\sqrt{5}\right)} \\
&= \frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}
\end{aligned}$$

4. Find the derivative of

$$\frac{1}{x}$$

at 2

Answer:

$$\begin{aligned}
f'(2) &= \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{2+h}-\frac{1}{2}\right) \\
&= \lim_{h\to 0}\frac{1}{h}\left(\frac{2}{2(2+h)}-\frac{2+h}{2(2+h)}\right) \\
&= \lim_{h\to 0}\frac{1}{h}\left(\frac{2-(2+h)}{2(2+h)}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{-h}{2(2+h)}\right) = \lim_{h\to 0}\frac{-1}{2(2+h)} \\
&= \frac{-1}{4}
\end{aligned}$$