Lagrange Interpolation

1. Suppose that Amanda owns a pool cleaning business. We know that she can clean 2 pools in 8 hours and 4 pools in 12 hours. Using Lagrange Interpolation on two points, create a function to model how fast she can clean pools.

   Answer:

   Let the independent variable x be the number of pools, and the dependent variable y be the length of time to clean them.

   Then we have two data points $(2, 8)$ and $(4, 12)$. Using the formula for Lagrange interpolation on two points we get:

$$\begin{aligned} f(x) &= 8\frac{x-4}{2-4} + 12\frac{x-2}{4-2} \\ &= 8\frac{x-4}{-2} + 12\frac{x-2}{2} \\ &= -4(x-4) + 6(x-2) \\ &= -4x + 16 + 6x - 12 \\ &= 2x + 4 \end{aligned}$$

   And so the model is

$$f(x) = 2x + 4$$

2. Using the model you just got, how long does it take her to clean 10 pools. Does this seem like a reasonable model? Are there any values that produce unusual results?

   Answer:

   The models gives $f(10) = 2(10) + 4 = 20 + 4 = 24$. Thus the models predicts that Amanda will take 24 hours to clean 10 pools.

   There are many things that could be pointed out here. One such example, is that $f(0) = 4$. We would expect to see $f(0) = 0$.

   This could be taken to mean that the model is not a good model, or an acknowledgement that models sometimes don't work as well near extreme values.

3. Suppose we also know that Amanda can clean 6 pools in 24 hours. Using Lagrange Interpolation on three points, create a function to model how fast she can clean pools.

Answer:

Let the independent variable x be the number of pools, and the dependent variable y be the length of time to clean them.

Then we have the data points $(2, 8)$,$(4, 12)$ and $(6, 24)$. Using the formula for Lagrange interpolation on three points we get:

$$\begin{aligned}
f(x) &= 8\frac{(x-4)(x-6)}{(2-4)(2-6)} + 12\frac{(x-2)(x-6)}{(4-2)(4-6)} + 24\frac{(x-2)(x-4)}{(6-2)(6-4)} \\
&= 8\frac{(x-4)(x-6)}{(-2)(-4)} + 12\frac{(x-2)(x-6)}{(2)(-2)} + 24\frac{(x-2)(x-4)}{(4)(2)} \\
&= 8\frac{(x-4)(x-6)}{8} + 12\frac{(x-2)(x-6)}{-4} + 24\frac{(x-2)(x-4)}{8} \\
&= (x-4)(x-6) - 3(x-2)(x-6) + 3(x-2)(x-4) \\
&= (x^2 - 10x + 24) - 3(x^2 - 8x + 12) + 3(x^2 - 6x + 8) \\
&= x^2 - 10x + 24 - 3x^2 + 24x - 36 + 3x^2 - 18x + 24 \\
&= x^2 - 4x + 12
\end{aligned}$$

And so the model is

$$f(x) = x^2 - 4x + 12$$

4. Using the model you just got, how long does it take her to clean 10 pools. Does this seem like a reasonable model? Are there any values that produce unusual results?

Answer:

The models gives $f(10) = 10^2 - 4(10) + 12 = 100 - 40 + 12 = 72$. Thus the models predicts that Amanda will take 72 hours to clean 10 pools.

There are many things that could be pointed out here. One such example, is that $f(1) = 9$, but $f(2) = 8$. Thus it takes less time to clean two pools then one pool.

This could be taken to mean that the model is not a good model, or an acknowledgement that models sometimes don't work as well near extreme values.