Average Rate of change and Function Classes

1. Draw a Venn diagram of the following Sets of functions:

   (a) Constant

   (b) Linear

   (c) Power

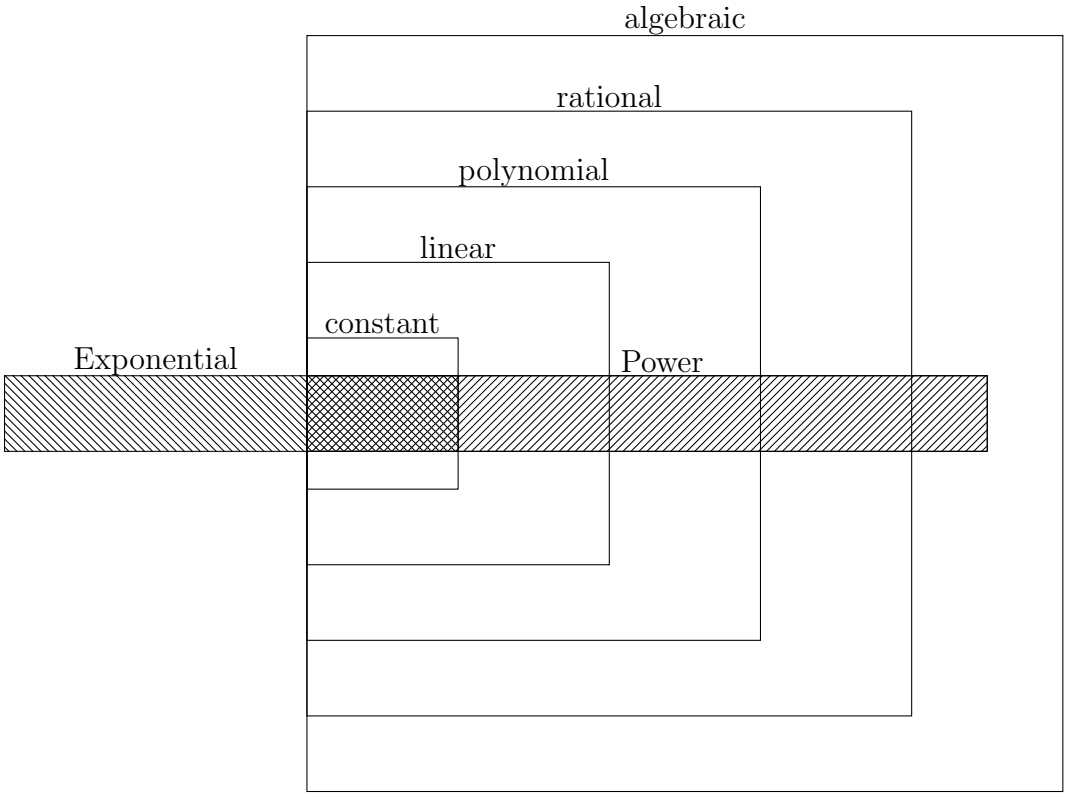
   (d) Polynomial

   (e) Rational

   (f) Algebraic

   (g) Exponential

   Answer:



2. Suppose we have a function defined on the interval from -1 to 6. If we know that the average rate of change of f on the interval from -1 to 1 is 5, and the average rate of change of f on the interval from 1 to 6 is -3, what is the average rate of change of f on the on the interval -1 to 6?

Answer:

The average rate of change of f over $[-1, 1]$ is $\frac{f(1)-f(-1)}{1-(-1)} = 5$. Thus $f(1) - f(-1) = 5(1 - (-1)) = 5(2) = 10$.

The average rate of change of f over $[1, 6]$ is $\frac{f(6)-f(1)}{6-1} = -3$. Thus $f(6) - f(1) = -3(6 - 1) = -3(5) = -15$.

Thus the average rate of change of f over $[-1, 6]$ is

$$\frac{f(6) - f(-1)}{6 - (-1)} = \frac{f(6) - f(1) + f(1) - f(-1)}{6 - (-1)} = \frac{10 + (-15)}{7} = -\frac{5}{7}$$

3. Suppose we have a function defined on the interval from a to b, and let c be a number such that $a < c < b$. If we know that the average rate of change of f on the interval from a to c is 3, and the average rate of change of f on the interval from c to b is 2, what is the average rate of change of f on the on the interval a to b? (**Hint: Your answer will be in terms of** $a, b,$ **and** c)

Answer:

The average rate of change of f over $[a, c]$ is $\frac{f(c)-f(a)}{c-a} = 3$. Thus $f(c) - f(a) = 3(c - a)$.

The average rate of change of f over $[c, b]$ is $\frac{f(b)-f(c)}{b-c} = 2$. Thus $f(b) - f(c) = 2(b - c)$.

Thus the average rate of change of f over $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{f(b) - f(c) + f(c) - f(a)}{b - a} = \frac{2(b - c) + 3(c - a)}{b - a} = \frac{-3a + 2b + c}{b - a}$$

4. Suppose we have a function defined on the interval from a to b, and let c be a number such that $a < c < b$. If we know that the average rate of change of f on the interval from a to c is r_1, and the average rate of change of f on the interval from c to b is r_2, what is the average rate of change of f on the on the interval a to b? (**Hint: Consider the quantities you know (i.e.** $a, b, c, r_1,$ **and** r_2**) and use the formula to solve for the one you don't.**)

Answer:

The average rate of change of f over $[a, c]$ is $\frac{f(c)-f(a)}{c-a} = r_1$. Thus $f(c) - f(a) = r_1(c - a)$.

The average rate of change of f over $[c, b]$ is $\frac{f(b)-f(c)}{b-c} = r_2$. Thus $f(b) - f(c) = r_2(b - c)$.

Thus the average rate of change of f over $[a, b]$ is

$$\frac{f(b) - f(a)}{b - a} = \frac{f(b) - f(c) + f(c) - f(a)}{b - a} = \frac{r_2(b - c) + r_1(c - a)}{b - a}$$