Math 1 Exam#2 Review

Dartmouth College

Thursday 10-13-16

Contents

If you are following along in the textbook, these are the sections that contain the material we covered.

Stewart's Calculus 8th Edition Sections for Exam#2:
1.2, 6.6, 11.1, 1.5, 1.6, 1.7, 1.8
Select Practice Problems from these sections:
Section 6.6 (Page #481): 1, 5-7, 11-13, 43-46
Section 11.1 (Page #744): 1-12, 13-18, 23-46, 49, 51, 53, 72-78
Section 1.5 (Page #59): 1-9, 11-12, 15-18, 29-34, 38-39, 46
Section 1.6 (Page #70): 1-4, 9-32, 37-39, 41, 50, 52, 59, 62-65
Section 1.7 (Page #81): 1-4
Section 1.8 (Page #91): 1-10, 17-32, 35-40, 45-47, 53-56, 71

Trigonometry

Stewart's Calculus 8th Edition Sections for Trigonometry:
1.2, 6.6

Select Practice Problems from these sections:
Section 6.6 (Page #481): 1, 5-7, 11-13, 43-46

For more trigonometry review see:
`https://math.dartmouth.edu/~m1f16/MATH1Docs/TrigonometryReview.pdf`
`https://math.dartmouth.edu/~m1f16/MATH1Docs/SullivanWS3.pdf`
`https://math.dartmouth.edu/~m1f16/MATH1Docs/SullivanWS3A.pdf`

Other topics on trigonometry we covered include:

- Finding periods and amplitudes of trigonometric functions.
- Graphs of $\sin(x), \cos(x), \tan(x)$ and their inverse functions.
- Graphs of functions of the form $A\sin(Bx + C) + D$ and $A\cos(Bx + C) + D$.

Slides from class:
```
https://math.dartmouth.edu/~m1f16/MATH1Docs/
09-30-Fri-Lecture-Slides-Musty.pdf
```

Sequences

Stewart's Calculus 8th Edition Sections for Sequences:
11.1
Select Practice Problems from these sections:
Section 11.1 (Page #744): 1-12, 13-18, 23-46, 49, 51, 53, 72-78

Other topics on sequences for the exam include:

- bounded + monotone $\implies$ convergent
- geometric sequences? what are they? do they converge? which ones converge? Let's find out!

Slides from class:
`https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-10-Slides-10-03-Mon.pdf`
`https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-11-Slides-10-05-Wed.pdf`
`https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-12-Slides-10-07-Fri.pdf`

Limits of functions

Stewart's Calculus 8th Edition Sections for limits of functions:
1.5, 1.6, 1.7
Select Practice Problems from these sections:
Section 1.5 (Page #59): 1-9, 11-12, 15-18, 29-34, 38-39, 46
Section 1.6 (Page #70): 1-4, 9-32, 37-39, 41, 50, 52, 59, 62-65
Section 1.7 (Page #81): 1-4

Other topics on limits of functions for the exam include:

- horizontal and vertical asymptotes
- write down a rational function with prescribed zeros and asymptotes

Slides from class:
`https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-12-Slides-10-07-Fri.pdf`
`https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-13-Slides-10-10-Mon.pdf`
`https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-13-Slides-10-10-Mon.pdf`

Continuity

Stewart's Calculus 8th Edition Sections for continuity:
1.8
Select Practice Problems from these sections:
Section 1.8 (Page #91): 1-10, 17-32, 35-40, 45-47, 53-56, 71

Other topics on continuity for the exam include:

- squeeze theorem

Slides from class:

https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-14-Slides-10-12-Wed.pdf
https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-Lecture-15-Slides-10-14-Fri.pdf

True/False

$\arccos(\cos(3\pi/4)) = 3\pi/4.$

True/False

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All trigonometric functions are continuous on their domains.

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$\lim_{x\to a^+} f(x) = L \implies \lim_{x\to a} = L$.

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Suppose $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x for all x. Use the Squeeze Theorem to evaluate $\lim_{x \to 1} g(x)$.

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Solution:

Since

$$\lim_{x\to 1} 2x = \lim_{x\to 1}(x^4 - x^2 + 2) = 2$$

we see by the Squeeze Theorem that $\lim_{x\to 1} g(x) = 2$.

Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x < 2 \\ 5 & \text{if } x = 2 \\ x - 4 & \text{if } x > 2 \end{cases}.$$

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Determine where g is continuous.

Let

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Solution:

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Does g have any discontinuities? If so, where and what type?

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g is continuous on the intervals $(-\infty, 1)$, $(1, 2)$, and $(2, \infty)$.

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Solution:

Jump at 1 and removable at 2.

Compute

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}.$$

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Solution:

The limit is 1. To see this, multiply numerator and denominator by $\sqrt{1+x}+\sqrt{1-x}$ and simplify.

Compute

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}.$$

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Solution:
Simplify the fractions to cancel h in the denominator limit is $-2/x^3$.

Let

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}.$$

For what values of the constant c is the function f continuous on $(-\infty, \infty)$?

Let

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For what values of the constant c is the function f continuous on $(-\infty, \infty)$?

Solution:

Since both defining functions are continuous (on all of $\mathbb{R}$), the only place we need to check for continuity is at 2. By the defining expressions we see that continuity at 2 depends only on the equality

$$c \cdot 2^2 + 2 \cdot 2 = 2^3 - c \cdot 2.$$

Since this equality is satisfied only by $c = 2/3$, then this is the unique value of c making f continuous everywhere.

Find all horizontal and vertical asymptotes of $f(x) = \tan x$.

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Solution:
f has no horizontal asymptotes. The vertical asymptotes are $x = \pm\pi/2, x = \pm 3\pi/2, x = \pm 5\pi/2, \ldots$.

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What about for $f^{-1}(x) = \arctan(x)$.

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What about for $f^{-1}(x) = \arctan(x)$.

Solution:
f^{-1} has no vertical asymptotes. It has 2 horizontal asymptotes $y = \pm\pi/2$.

Let $f(x) = \log(x)$. Where is f continuous? Is f discontinuous anywhere?

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What if instead we consider $f(x) = \log|x|$?

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f is continuous on $(0, \infty)$. f does not have any discontinuities.

What if instead we consider $f(x) = \log|x|$?

Solution: Well, what is the domain of f?

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What if instead we consider $f(x) = \log|x|$?

Solution: Well, what is the domain of f? Yep, $(-\infty, 0) \cup (0, \infty)$. Where is f continuous? It's continuous on its domain. Does f have any discontinuities? Yes. Now that f is defined near zero, f has an infinite discontinuity at zero. Should we graph f?

Let

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}.$$

Where is f continuous? Is f discontinuous anywhere?

Let

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}.$$

Where is f continuous? Is f discontinuous anywhere?

Solution: f is continuous everywhere.

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Solution:

Well, consider a rational function. We know we want the denominator to be zero at $x = -3, 0, 2$. So how about $f_1(x) = 1/((x+3)(x-2)x)$?

Find a function with horizontal asymptote $y = 5$ and vertical asymptotes $x = -3$, $x = 2$, $x = 0$.

Solution:

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Solution:

Well, consider a rational function. We know we want the denominator to be zero at $x = -3, 0, 2$. So how about $f_1(x) = 1/((x+3)(x-2)x)$? Well, that has the correct vertical asymptotes but what about the horizontal one? To yield the correct horizontal asymptote we define

$$f(x) = \frac{5(x-5)^3}{(x+3)(x-2)x}.$$

Why $x - 5$? Well, it doesn't really matter except that we don't want the numerator to be zero when the denominator is. So 5 could have been anything except $-3, 0, 2\ldots$