Math 1 Final Exam Review

Dartmouth College

The Last Week

Contents

Reminders/Announcements

- The final exam date is **Friday November 18th**
- The final exam time it **11:30am - 2:30pm**
- The final exam location is **Dartmouth Hall 105**

Functions

Topics Concerning Functions

- definitions of lots of types of functions
- even and odd functions
- continuity
- domain / range
- solve $f(x) = 0$
- inverse functions
- domain / range under transformations

Exercises Concerning Functions

1. Let $f(x) = \sin(x)$ and $g(x) = \arcsin(x)$. What are the domains and ranges of $f, g, f \circ g, g \circ f$?
2. Let $f(x) = x^2$ and $g(x) = \sqrt{x+1}$. What are the domains and ranges of $f, g, f \circ g, g \circ f$?
3. Find all solutions to $\ln(x+2) + \ln(x-2) = \ln(6)$.
4. Find all solutions to $2^{3x+1} = 4^x$.
5. Suppose $f(x)$ is one-to-one and has domain $[-2, 3]$ and range $[0, 8]$. Find the domain and range of $-f(-x-1)$.
6. Suppose $f(x)$ is one-to-one and has domain $[-2, 3]$ and range $[0, 8]$. Find the domain and range of $3f(2x+1)$.
7. Suppose $f(x)$ is one-to-one and has domain $[-2, 3]$ and range $[0, 8]$. Find the domain and range of $4f^{-1}(-x) + 1$.

More Exercises Concerning Functions

1. What is the definition of continuity? (It involves a limit)
2. Which trigonometric functions are continuous on $\mathbb{R}$?
3. Is $f(x) = \frac{x^2+x}{x^4}$ an even function?
4. Find all solutions to $\sin(x) = 0$.
5. Find all solutions to $\cos\left(x - \frac{\pi}{2}\right) = 0$.
6. Find all solutions to $\ln(x) + \ln(x-1) = 1$.
7. Find all solutions to $\ln(2x+1) = 2 - \ln(x)$.
8. Find all solutions to $2^{x-5} = 3$.
9. Find all solutions to $e^{7-4x} = 6$.

Sequences

Topics Concerning Sequences

- computing terms of a sequence
- increasing / decreasing
- bounded
- convergence
- geometric sequences

Exercises Concerning Sequences

Consider the sequences $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}, \{c_n\}_{n=0}^{\infty}$ defined by

$$a_n = (-1)^n, \quad b_n = \left(\frac{2}{5}\right)^n, \quad c_n = \frac{3n^2 + 5n}{\sqrt{7n^4 + 2}}$$

1. Is $\{b_n\}_{n=0}^{\infty}$ increasing, decreasing, or neither?
2. Does $\{a_n\}_{n=0}^{\infty}$ converge? If so, to what?
3. Does $\{a_n b_n\}_{n=0}^{\infty}$ converge? If so, to what?
4. Does $\{c_n\}_{n=0}^{\infty}$ converge? If so, to what?
5. How many of these sequences are bounded?
6. Is $\{a_n b_n\}_{n=0}^{\infty}$ bounded? If so, by what?

More Exercises Concerning Sequences

1. What is a geometric sequence?
2. How can you tell if a geometric sequence converges?
3. What does it mean for a sequence to be bounded?
4. Are all bounded sequences convergent?
5. Are all convergent sequences bounded?

Trigonometry

Topics Concerning Trigonometry

- computing special values
- computing special inverse trig values
- drawing triangles to compute values and simplify expressions
- graphing $\sin(x)$, $\cos(x)$, $\tan(x)$ and their transformations
- graphing $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$

Exercises Concerning Trigonometry

1. Simplify the expression $\tan(\arccos(1/\sqrt{2}))$.
2. Simplify the expression $\cos(\arcsin(\sqrt{3}/2))$.
3. Simplify the expression $\tan(\operatorname{arcsec}(x))$.
4. Simplify the expression $\tan(\arccos(x/(x+1)))$.
5. Graph the function $3\sin(2(x-1))+3$.

More Exercises Concerning Trigonometry

1. https://math.dartmouth.edu/~m1f16/MATH1Docs/TrigonometryReview.pdf

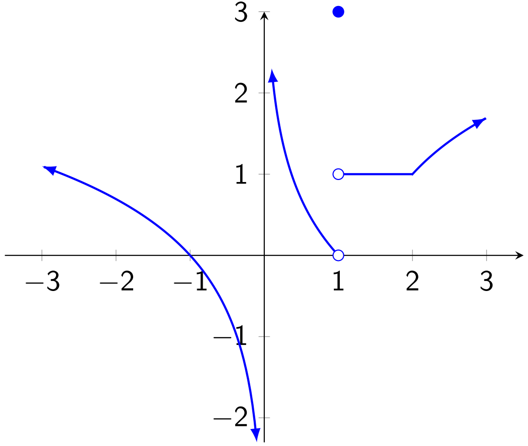
Limits

Topics Concerning Limits

- compute limits using graphs
- compute limits using algebraic manipulations
- compute limits in the definition of the derivative

Exercises Concerning Limits

Computing limits using the graph of a function. . .



Exercises Concerning Limits

1. Compute $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h}$.
2. Compute $\lim_{t\to \pi/2} \left(\left(t-\frac{\pi}{2}\right)\sin(t)\right)$.
3. Let $f(x) = x^2 - x$. Compute $f'(x)$ using the limit definition.
4. Let $f(x) = \frac{x}{x-1}$. Compute $f'(x)$ using the limit definition.

More Exercises Concerning Limits

1. Compute $\lim_{h\to 0} \frac{(-5+h)^2-25}{h}$
2. Compute $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$
3. Compute $\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+1}\right)$

Derivatives

Topics Concerning Derivatives

- use the limit definition to compute derivatives
- use derivative rules to compute derivatives
- use implicit differentiation to find derivatives
- use derivatives to find tangent lines

Exercises Concerning Derivatives

1. Let $f(x) = x^2 - x$. Compute $f'(x)$ using the limit definition.
2. Let $f(x) = \frac{x}{x-1}$. Compute $f'(x)$ using the limit definition.
3. Compute $\frac{d}{dx}(x - e^7)$.
4. Compute $\frac{d}{dx}(x^5 - x^2 + 3)$.
5. Compute $\frac{d}{dx}\left(\frac{\tan(x)}{1+\cos(x)}\right)$.
6. Find $\frac{dy}{dx}$ for the curve $x^2 + xy + y^2 = 0$.
7. Find $\frac{dy}{dx}$ for the curve $\cos(x + y) = x^2 + 3y$.
8. Find $\frac{dy}{dx}$ for the curve $\ln(y) = y^3 + x^3$.

Exercises Concerning Derivatives

1. Suppose f^{-1} is the inverse of a differentiable function f and $f(4) = 5$, $f'(4) = 2/3$. Find $(f^{-1})'(5)$.
2. Let $f(x) = x^3 + 3\sin(x) + 2\cos(x)$ and $a = 2$. Find $(f^{-1})'(a)$.
3. Compute $\frac{d}{dx}\left(\arctan\sqrt{x^2+5x}\right)$.
4. Compute $\frac{d}{dx}\left(\log_2\left(\frac{2x+1}{3x^2-1}\right)\right)$.
5. Compute $\frac{d}{dx}\left(e^{\sqrt{x^3+3x^2-2x}}\right)$.
6. Compute $\frac{d}{dx}\left(3^{7x+1}\right)$.
7. Compute $\frac{d}{dx}\left(x^3\arcsin(3x^2)\right)$.

More Exercises Concerning Derivatives

1. Find the derivative of $a(x) = \frac{(x^3+5x+1)\log(x^2)}{e^{2x}}$.
2. Find the derivative of $b(x) = \sin\left(\frac{2^x}{x^3+x}\right)$.
3. Find the derivative of $c(x) = e^{x^2 \cdot \cos(x)}$.
4. Find the derivative of $d(x) = \frac{\ln(x^3+3x^2+3x+1)}{\sin(x)}$.
5. Find the derivative of $f(x) = \ln(13x+5)$.
6. Find the derivative of $g(x) = e^x \cos(x)$.

More Exercises Concerning Derivatives

1. Find an equation of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(2, 1)$.
2. Find an equation of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(-1, 1)$.
3. Find an equation of the tangent line to the curve $x^2 + 2xy + 4y^2 = 12$ at the point $(2, 1)$.

Applications

Topics Concerning Applications

- Newton's method
- linearization
- Taylor polynomials

Exercises Concerning Applications

1. Let $f(x) = x^4 - 5x^2 + 2x - 1$. Write a formula for x_{n+1} in terms of x_n using Newton's method. Simplify as much as possible.
2. Let $f(x) = x^4 - 5x^2 + 2x - 1$ and $x_0 = 3$. Compute x_1 using Newton's method.
3. Let $f(x) = \sin(x)$. Compute the linearization $L(x)$ at $a = 0$.
4. Let $f(x) = \sin(x)$. Compute the degree 3 Taylor polynomial centered at 0.

More Exercises Concerning Applications

1. Given a function $f(x)$ and a starting value x_0, state the formula to obtain x_1 via Newton's method. What is the formula for x_{n+1} in terms of x_n?
2. Given a function $f(x)$ and a real number a, state the formula for the degree 3 Taylor polynomial of f centered at a.
3. Compute the degree 3 Taylor polynomial of $f(x) = x^3 - x$ centered at 0.
4. Compute the degree 4 Taylor polynomial of $f(x) = e^x$ centered at 0.

True/False

True/False

1. $\sin(x)$ is an even function.
2. $\sin(x)$ is an odd function.
3. e^x is an increasing function.
4. $\ln(x)$ is a decreasing function.
5. If $f(x) = (x-5)^2 + 3$, then f is one-to-one on $[-1, 5]$.
6. If $f(x) = (x-5)^2 + 3$, then f is one-to-one on $[0, 7]$.

True/False

1. The sequence $\{a_n\}_{n=0}^{\infty}$ defined by
$$a_n = (-1)^n \left(\frac{7}{9}\right)^n$$
is bounded.
2. The sequence $\{a_n\}_{n=0}^{\infty}$ defined by
$$a_n = (-1)^n \left(\frac{7}{9}\right)^n$$
is decreasing.
3. The sequence $\{a_n\}_{n=0}^{\infty}$ defined by
$$a_n = (-1)^n \left(\frac{7}{9}\right)^n$$
converges.

True/False

1. $\arctan(\cos(\pi/2)) = 0$
2. $\arccos(\sin(2\pi/3)) = \pi/6$
3. $\arctan(-1) = \pi/4$
4. $\arccos(\sin(\pi/3)) = \pi/6$
5. $\arcsin(\sin(\pi)) = \pi$