Math 1 Lecture 25

Dartmouth College

Monday 11-07-16

Contents

Reminders/Announcements

- Last Quiz today!
- Last written HW due Wednesday!
- WebWork due Wednesday... but not the last one 😇

This Week

- Linear Approximations (Monday)
- Taylor Polynomials (Wednesday)
- Taylor Polynomial Approximations (Friday)

Approximating Functions

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Sounds simple. . .

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More formally, given a function f and a real number a, we define the **linearization of f at a** by the linear function:

$$\boxed{L(x) = f(a) + f'(a)(x - a)}.$$

Example

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and for x near 1 we can approximate $f(x)$ with $L(x)$. Specifically,

$$f(0.98) = \sqrt{3.98} \approx L(0.98) = 2 + \frac{1}{4}(0.98-1) = 1.995$$
$$f(1.05) = \sqrt{4.05} \approx L(1.05) = 2 + \frac{1}{4}(1.05-1) = 2.0125.$$

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So when x is near 0, $\sin(x) \approx x$. This approximation is used in deriving the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for the period of a pendulum of length L.

Examish Exercises

1. Find the linearization of f at a:
   (a) $f(x) = x^3 - x^2 + 3$, $a = -2$
   (b) $f(x) = \sin(x)$, $a = \pi/6$
   (c) $f(x) = \sqrt{x}$, $a = 4$
   (d) $f(x) = \frac{2}{\sqrt{x^2-5}}$, $a = 3$
2. Use the linearization of f at a to approximate:
   (a) $(1.999)^4$
   (b) $\sqrt[3]{1001}$
   (c) $\frac{1}{4.002}$
   (d) $\sqrt{100.5}$