Math 1 Lecture 23

Dartmouth College

Wednesday 11-02-16

Contents

Reminders/Announcements

- WebWork due Friday
- Written HW due today
- x-hour tomorrow:
  - proofs of inverse trig derivatives
  - examish exercises

Last Time

- Implicit differentiation
- $(f^{-1})'(a) = 1/f'(f^{-1}(a))$
- Derivatives of inverse trigonometric functions

$\frac{d}{dx}(\arcsin x)$ revisited

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Thus

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The domain of this function is $(0, \infty)$. Do you see why?

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The only difference is that this derivative is valid on $(-\infty, 0) \cup (0, \infty)$. Consider the graphs of $\ln(x)$ and $\ln|x|$...

$\frac{d}{dx}(\log_a x)$

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First note that

$$\log_a(x) = \frac{\ln(x)}{\ln(a)} = \underbrace{\frac{1}{\ln(a)}}_{\text{constant}} \cdot \ln(x).$$

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$$\begin{aligned}\frac{d}{dx}(\log_a(x)) &= \frac{d}{dx}\left(\frac{\ln(x)}{\ln(a)}\right)\\ &= \frac{1}{\ln(a)}\frac{d}{dx}(\ln(x))\\ &= \frac{1}{\ln(a)} \cdot \frac{1}{x}\\ &= \frac{1}{x\ln(a)}.\end{aligned}$$

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If $a = e$ does this check out?

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Examish Exercises

Find the derivative of each function and the domain on which it is valid.

1. $y = \ln(x + 5)$
2. $y = \ln|x + 5|$

Examish Exercises

1. $f(x) = x \ln x - x$
2. $f(x) = \sin(\ln x)$
3. $y = \ln \frac{1}{x}$
4. $g(x) = \ln(xe^{-2x})$
5. $f(x) = \log_{10} x$
6. $h(x) = \log_{10} \sqrt{x}$
7. $y = 2^x$
8. $y = 5^{2x+1}$
9. $y = (x^2 + 2)^2(x^4 + 4)^4$
10. $y = (2x + 1)^x$