Math 1 Lecture 20

Dartmouth College

Wednesday 10-26-16

Contents

Reminders/Announcements

- WebWork due Friday
- x-hour problem session drop in Thursday
- Quiz Monday
- Turn in "MidQuarter" surveys
- Turn in Written Homework
- Office hours today 1pm - 3pm

Last Time

- Derivatives of products and quotients
- Higher derivatives

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Let's look at a couple practice problems from last time...

For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent line?

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Also, compute

$$\frac{d^2}{dx^2}(f(x)), \quad \frac{d^3}{dx^3}(f(x)), \text{ and } \quad \frac{d^4}{dx^4}(f(x)).$$

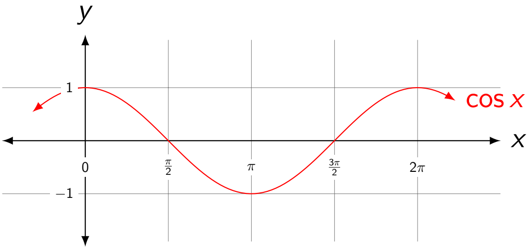
Suppose $h(2) = 4$, $h'(2) = -3$, and $h''(2) = 1$. Compute the following values:

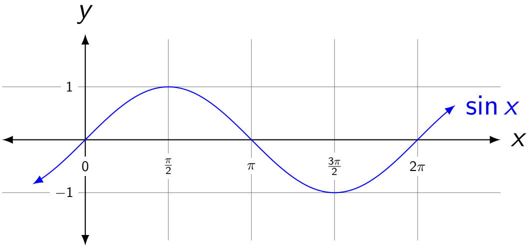
1.

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Bigg|_{x=2}$$

2.

$$\frac{d^2}{dx^2}\left(\frac{h(x)}{5}\right)\Bigg|_{x=2}$$


y
x
1
−1
0
π/2
π
3π/2
2π
cos x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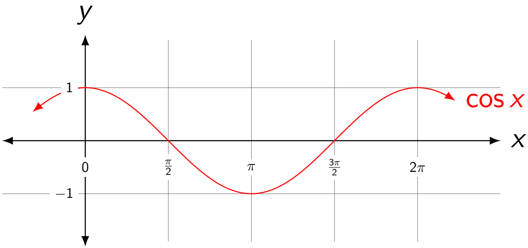
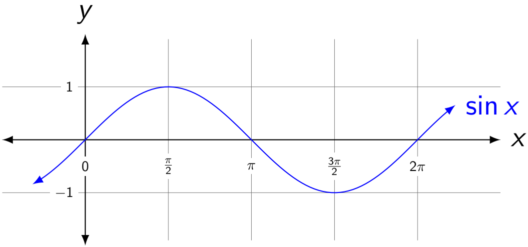
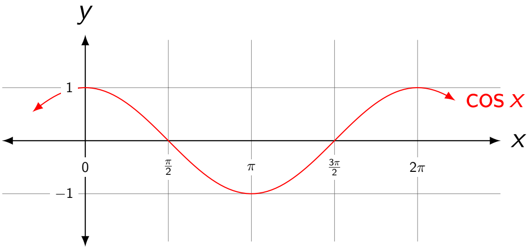
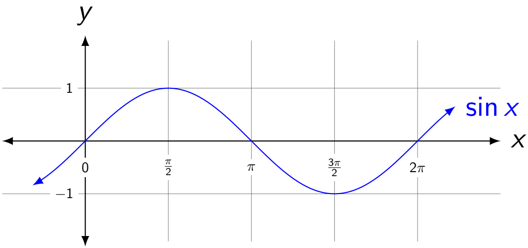


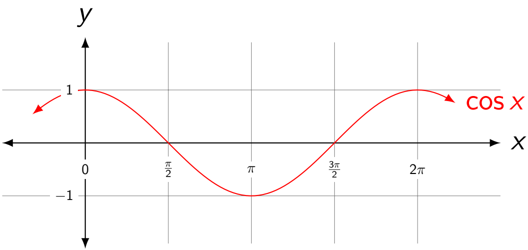


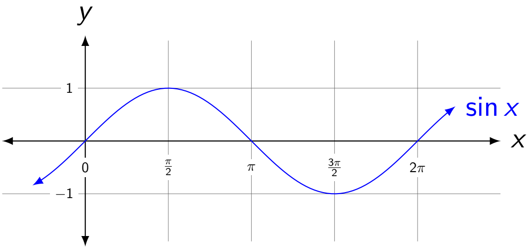
Let $f(x) = \sin(x)$.



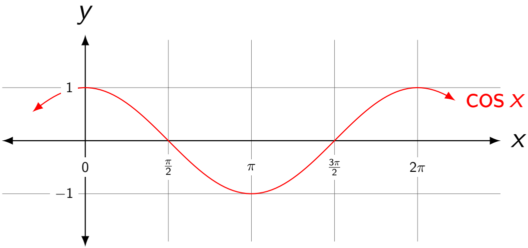


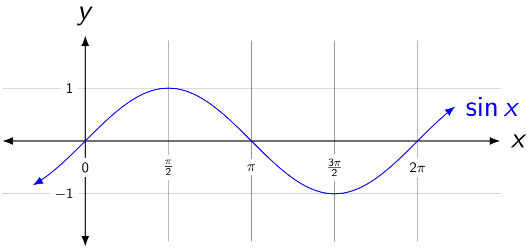
Let $f(x) = \sin(x)$. What is $f'(0)$?





Let $f(x) = \sin(x)$. What is $f'(0)$? What is $f'(\pi/2)$?





Let $f(x) = \sin(x)$. What is $f'(0)$? What is $f'(\pi/2)$? How does this value relate to the function $\cos(x)$?

By observations on the previous slide we might guess that

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$
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It is beyond the scope of this course to verify these via the definition, but we do need to be able to use these rules to compute examples.

Note that we can compute the derivatives of all 6 trigonometric functions using the previous slide and the quotient rule.

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$$\begin{aligned}
\frac{d}{dx}(\sin x) &= \cos x \\
\frac{d}{dx}(\cos x) &= -\sin x \\
\frac{d}{dx}(\tan x) &= (\sec x)^2 \\
\frac{d}{dx}(\cot x) &= -(\csc x)^2 \\
\frac{d}{dx}(\sec x) &= \sec x \tan x \\
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\end{aligned}$$

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As an exercise let's verify a few of these!

Computational Examples

1. $f(x) = x^2 \sin x$
2. $f(\theta) = \dfrac{\sin\theta}{1 + \cos\theta}$
3. $s(t) = e^t \sin t$
4. $y = \dfrac{t \sin t}{1 + t^2}$

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Solution:
$f'(x) = 1 + (\sec x)^2$ and $f'(\pi) = 2$. Thus we are looking for a line through the point (π, π) with slope 2. Such a line is given by

$$y - \pi = 2(x - \pi).$$

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First compute y' and y'' and collect terms in the expression for $y'' + y' - 2y$. Then compare coefficients to get 2 equations involving the unknown values A and B. Solve one equation for one of the unknowns in terms of the other one. Substitute into the other equation. Then you win. We find that $A = -3/10$ and $B = -1/10$.