Math 1 Lecture 17

Dartmouth College

Wednesday 10-19-16

Contents

Reminders/Announcements

- Exam#2 is Thursday 10/20/16 and will cover material from Trigonometry up to and NOT including derivatives
- Exam review during x-hour 10/20/16
- Exam Review Slides:
  `https://math.dartmouth.edu/~m1f16/MATH1Docs/Musty-x-hour-Slides-10-13-Thur.pdf`
- Because of the exam there will be no WebWork due Friday 10/21/16

Last time

- The derivative at a point
- The derivative as an instantaneous rate of change
- The derivative as the slope of a tangent line

Suppose the function $f(x)$ has a tangent line at the point $(4, 3)$ (i.e. $f(4) = 3$) passes through the point $(0, 2)$. Find $f'(4)$.

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Solution: $f'(4) = 1/4$.

Write the following limit as $f'(a)$ for some f and some a.

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$$\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}.$$

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Solution:

$$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} = f'(a)$$

for $f(x) = \sqrt{x}$ and $a = 9$.

The derivative as a function

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

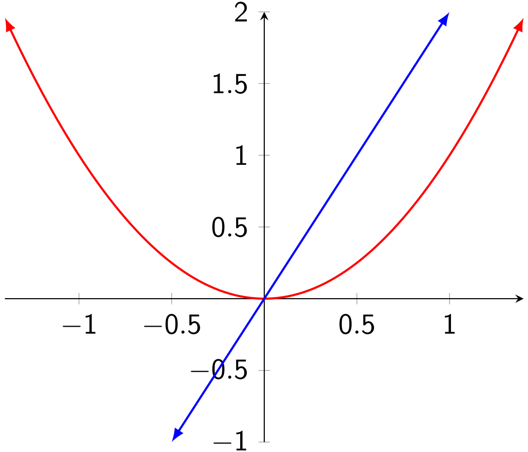
The domain of this new function $f'(x)$ is precisely the numbers a in the domain of f where the number $f'(a)$ is defined. Other notations for $f'(x)$ include

$$\frac{dy}{dx}, \frac{d}{dx}(f(x)), D_x(f), \ldots$$

Examples

- Let $f(x) = \sqrt{x}$. Show that $f'(x) = \frac{1}{2\sqrt{x}}$ using the definition.
- Let $f(x) = x^2$. Show that $f'(x) = 2x$ using the definition.
- Let $f(x) = x^3 - x$. Show that $f'(x) = 3x^2 - 1$ using the definition.

Here are the graphs $f(x)$ and $f'(x)$ for some function f...



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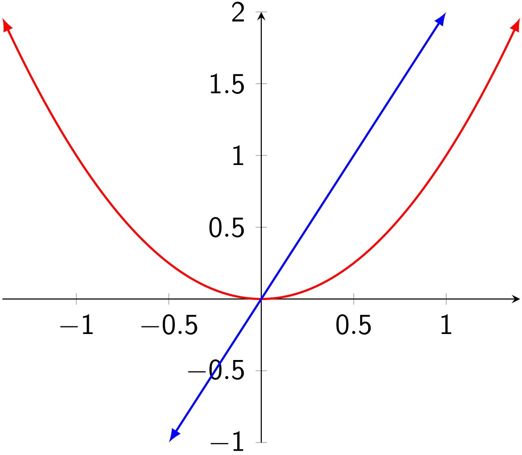

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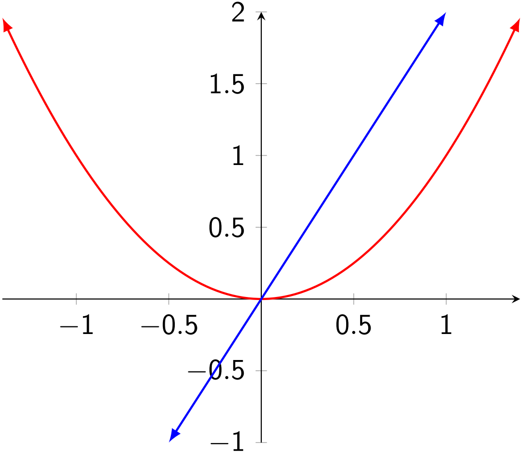

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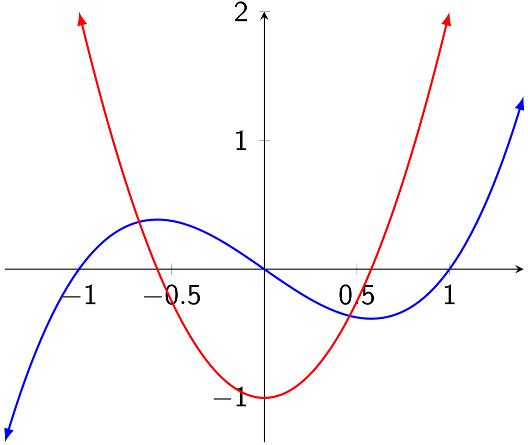

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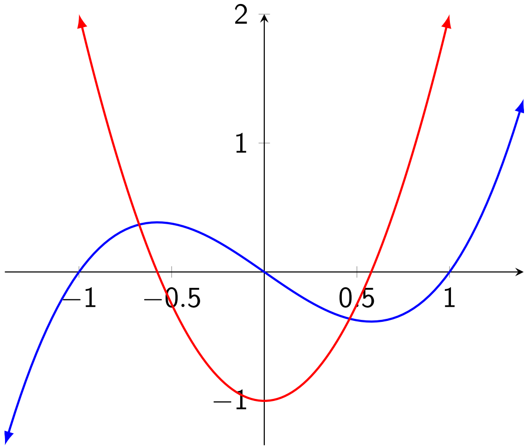

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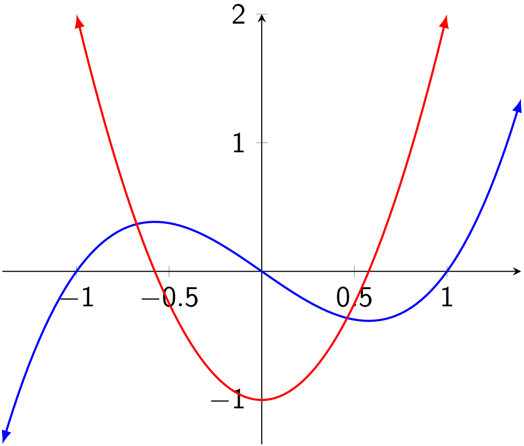

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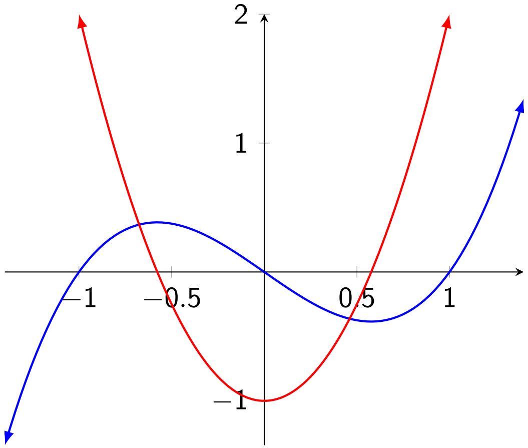

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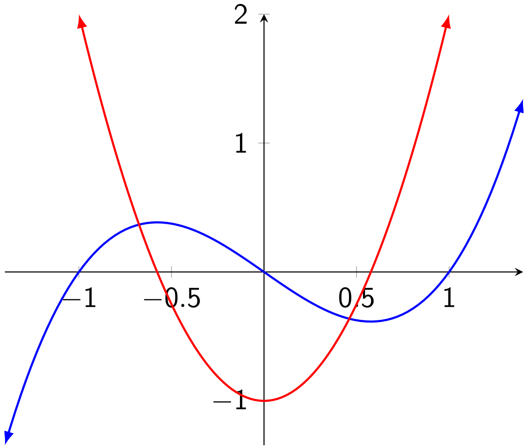

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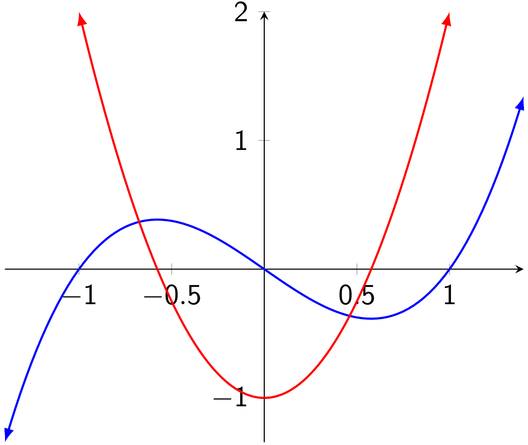

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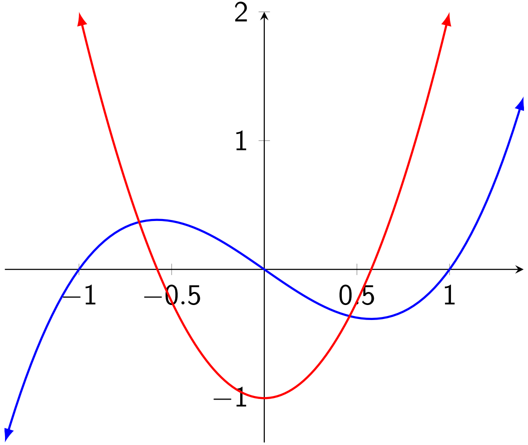

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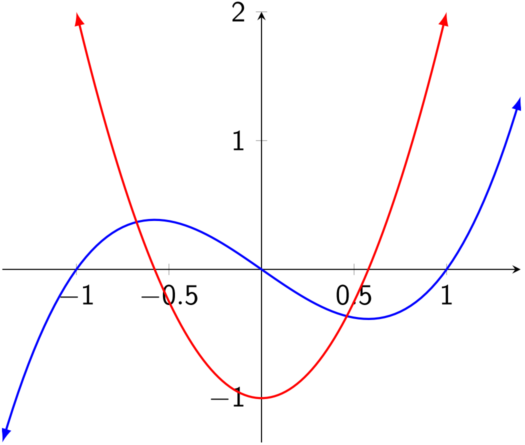

What does f' tell us about how f is increasing or decreasing (from left to right)? Where is f' equal to zero? Can we find the precise intervals where f is increasing?

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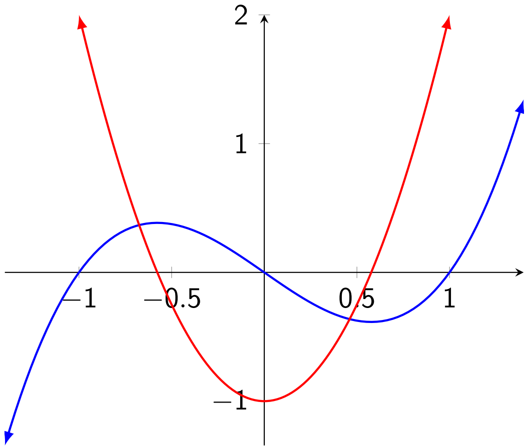

What does f' tell us about how f is increasing or decreasing (from left to right)? Where is f' equal to zero? Can we find the precise intervals where f is increasing? Let's find out!

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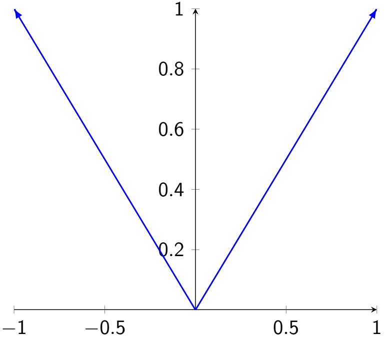

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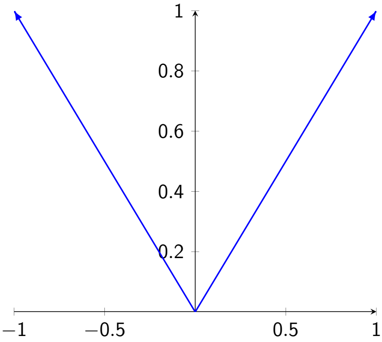

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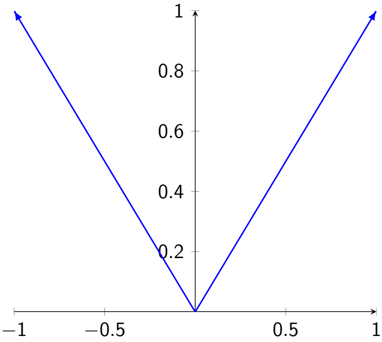

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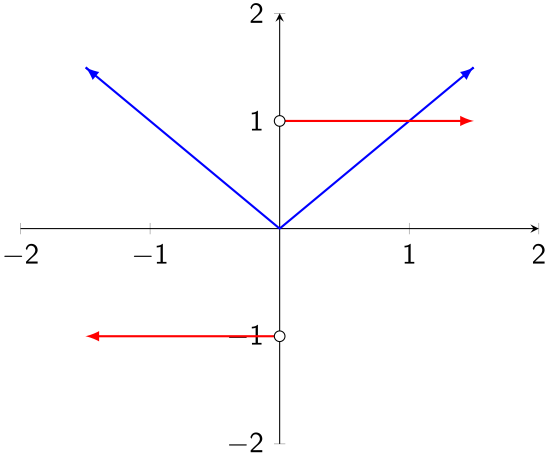
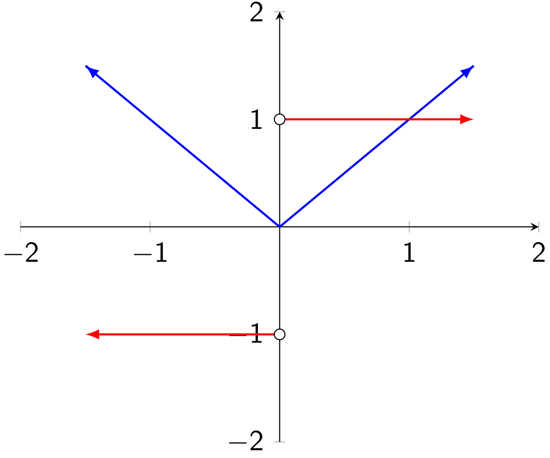
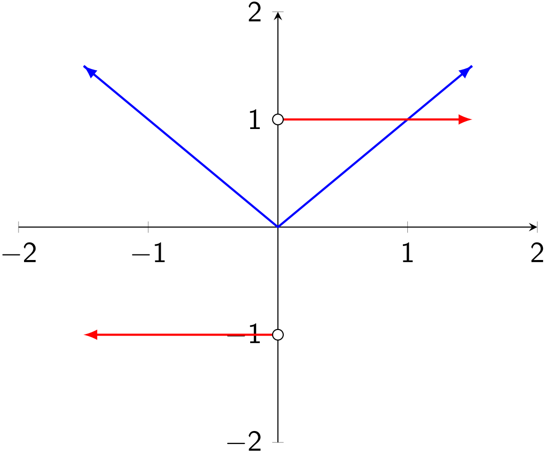
What does the graph of the derivative look like?


2
1
−1
−2
−2
−1
1
2



The derivative of $f(x) = |x|$ is not continuous at 0!



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