Math 1 Lecture 12

Dartmouth College

Friday 10-07-16

Contents

Reminders/Announcements

- WebWork due Monday
- Quiz Monday
- Written Homework due Wednesday

Geometric Sequences

A sequence is **geometric** if it is of the form

$$\{a \cdot r^n\}_{n=0}^{\infty}$$

with a, r in $\mathbb{R}$. We call r the **common ratio**.

Consider the sequence

$$\{2 \cdot 3^n\}_{n=0}^{\infty}$$

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Monotone? Increasing.
Bounded? No.
Convergent? No.

Consider the sequence

$$\left\{2 \cdot \left(\frac{1}{3}\right)^n\right\}_{n=0}^{\infty}$$

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Monotone? Yes. Decreasing.
Bounded? Yes. By the first term.
Convergent? Yep. It converges to zero.

For what values of r does a geometric sequence converge?

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A geometric sequence converges if and only if r in $(-1, 1]$.

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Fixed a typo here from last time

Back to the definition

Consider the sequence

$$\{0.9, 0.99, 0.999, 0.9999, \dots\}$$

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It is *clear* that this sequence converges to 1. But how do we *prove* it? Suppose $\varepsilon = 0.1$. Can we find N such that $|a_n - L| < \varepsilon$ for all $n \geq N$?

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It is *clear* that this sequence converges to 1. But how do we *prove* it? Suppose $\varepsilon = 0.1$. Can we find N such that $|a_n - L| < \varepsilon$ for all $n \geq N$? Sure! Take $N = 2$. What if $\varepsilon = 0.01$?

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It is *clear* that this sequence converges to 1. But how do we *prove* it? Suppose $\varepsilon = 0.1$. Can we find N such that $|a_n - L| < \varepsilon$ for all $n \geq N$? Sure! Take $N = 2$. What if $\varepsilon = 0.01$? Can we find N then? Yes! So what?

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A sequence is convergent if we can *always* find an N no matter how small we choose $\varepsilon > 0$.

More Examples of Sequences

Consider the sequence

$$\left\{\tan\left(\frac{\pi}{2} - \frac{1}{n}\right)\right\}_{n=1}^{\infty}$$

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Monotone? Decreasing.
Bounded? By the first term...
Convergent? Yes. Notice that

$$\begin{aligned} a_n &= (\sqrt{n+1}-\sqrt{n})\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} \\ &= \frac{1}{\sqrt{n+1}+\sqrt{n}} \end{aligned}$$

which converges to zero.

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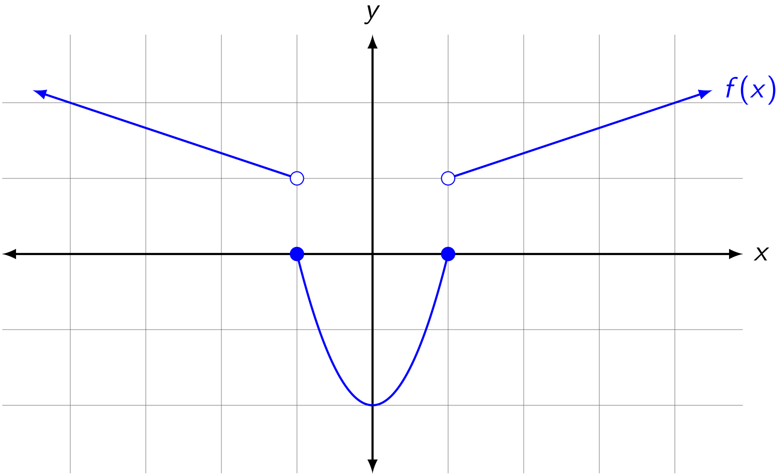
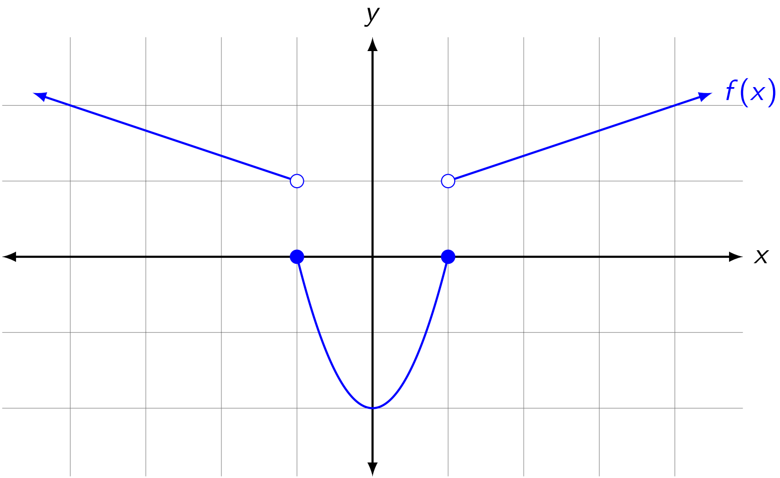
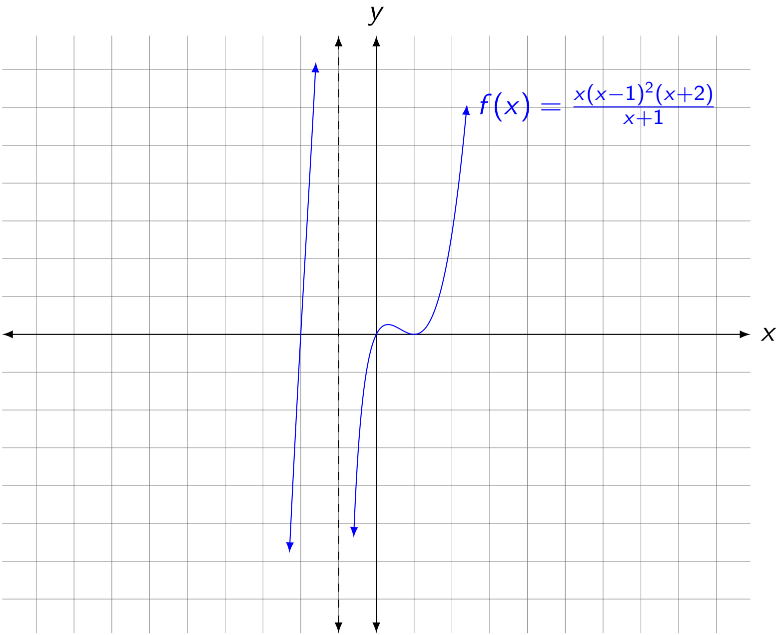

y
x
f(x)



Write out $f(x)$ as a function defined in "pieces".

For the function $f(x)$ given on the previous slide we have

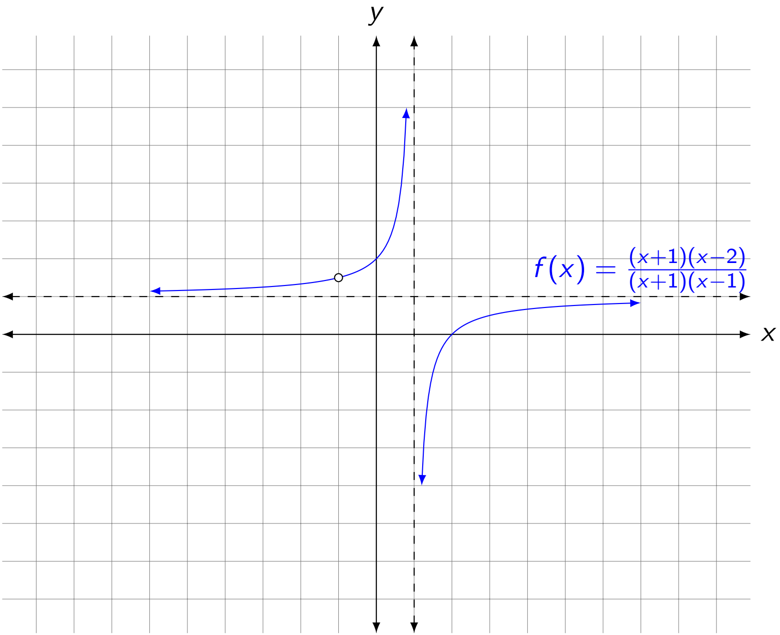
$\lim_{x\to\infty} f(x) = \infty$
$\lim_{x\to-\infty} f(x) = \infty$
$\lim_{x\to-1^-} f(x) = 1$
$\lim_{x\to-1^+} f(x) = 0$
$\lim_{x\to1^-} f(x) = 0$
$\lim_{x\to1^+} f(x) = 1$
$f(-1) = f(1) = 0.$



y
x
$f(x) = \frac{x(x-1)^2(x+2)}{x+1}$

For the function $f(x)$ given on the previous slide we have

$\lim_{x \to \infty} f(x) = \infty$
$\lim_{x \to -\infty} f(x) = -\infty$
$\lim_{x \to -1^-} f(x) = \infty$
$\lim_{x \to -1^+} f(x) = -\infty$



y
x
f(x) = (x+1)(x−2) / (x+1)(x−1)

For the function $f(x)$ given on the previous slide we have

$\lim_{x\to\infty} f(x) = 1$
$\lim_{x\to-\infty} f(x) = 1$
$\lim_{x\to-1^-} f(x) = 3/2$
$\lim_{x\to-1^+} f(x) = 3/2$
$\lim_{x\to-1} f(x) = 3/2$
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Note that limits can tell us about the behavior of a function even when the function is undefined!

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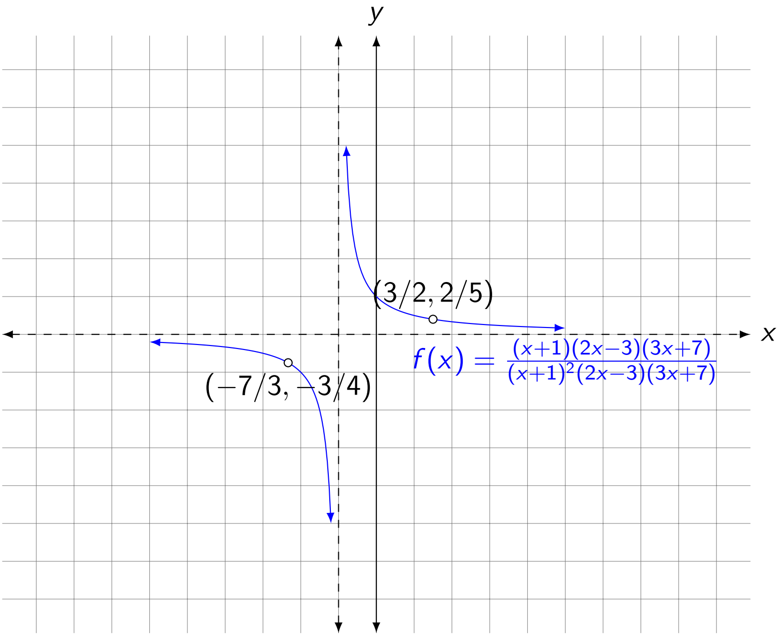
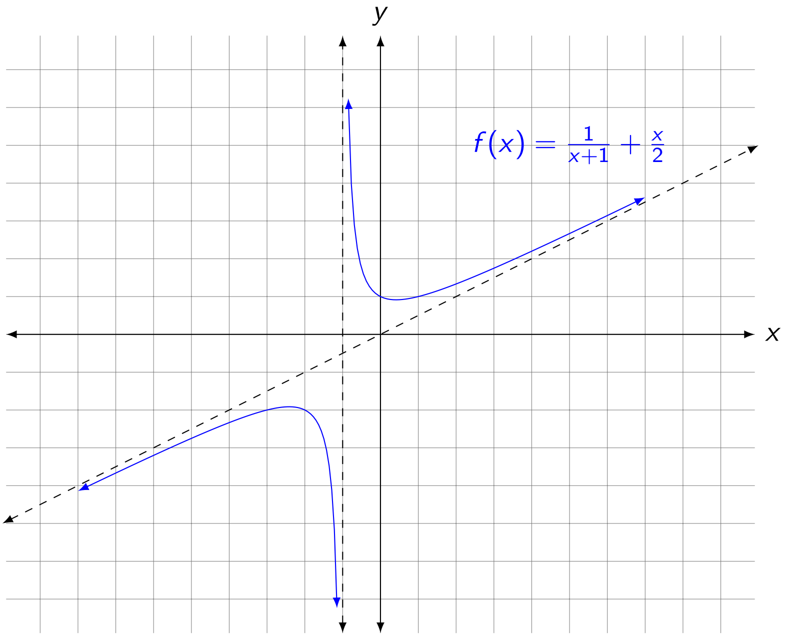
Note that limits can tell us about the behavior of a function even when the function is undefined! Huh?



y
x
(3/2, 2/5)
(−7/3, −3/4)
$f(x) = \frac{(x+1)(2x-3)(3x+7)}{(x+1)^2(2x-3)(3x+7)}$

For the function $f(x)$ given on the previous slide we have

$\lim_{x\to -7/3} f(x) = -3/4$
$\lim_{x\to 3/2} f(x) = 2/5$
$\lim_{x\to -1^-} f(x) = -\infty$
$\lim_{x\to -1^+} f(x) = \infty$
$\lim_{x\to 1} f(x)$ "does not exist".



y
x
f(x) = 1/(x+1) + x/2

For the function $f(x)$ given on the previous slide we have

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Sometimes the limit is just the value of the function. This is what it means for a function to be "continuous" as we will see later.

Asymptotes

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A function $f(x)$ has a **vertical asymptote** $x = a$ (a in $\mathbb{R}$) if $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.

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A function $f(x)$ has a **horizontal asymptote** $y = L$ (L in $\mathbb{R}$) if $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} = L$.

More Examples as Time Permits

$\lim_{x \to -4} \frac{x^2+10x+24}{x+4} =$

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