Math 1
1st Midterm

September 29, 2016

Name (in block capital letters):

Instructor (tick one box):

☐ **Section 1: M. Musty(10:10am)**

☐ **Section 2: E. Sullivan(11:30am)**

☐ **Section 3: A. Babei(12:50pm)**

☐ **Section 4: M. Dennis(2:10pm)**

Instructions: You are not allowed to provide or receive help of any kind (closed book examination). However, you may ask the instructor for clarification on problems.

1. **Wait** for **signal** to begin.
2. **Write** your **name** in the space provided, and **tick one box** to indicate which section of the course you belong to.
3. Calculators, computers, cell phones, or other computing devices are **not allowed**. In consideration of other students, please **turn off cell phones** or other electronic devices which may be disruptive.
4. Unless otherwise stated, you must **justify your solutions** to receive full credit. Work that is illegible may not be graded. Work that is scratched out will not be graded.

Problem	Score	Possible
1		10
2		20
3		15
4		12
5		12
6		10
7		7
Total		86

1. (**10 points**) Which of the following statements are always true? Write "**T**" for true and "**F**" for false. Your computations will not be graded on this problem.

(a) $\square$ Given the sequence $\left\{\frac{(-1)^n}{n^2+1}\right\}_{n=1}^{\infty}$, the third term is $\frac{-1}{10}$.

(b) $\square$ Recall that a sequence $\{a_n\}$ is bounded if there exists a number M such that $|a_n| \leq M$ for all n. The sequence $\{\frac{1}{n}\}_{n=1}^{\infty}$ is bounded by zero.

(c) $\square$ The function $f(x) = \frac{x^3}{x^2-1}$ is an even function.

(d) $\square$ The function $f(x) = \frac{x^4}{x+1}$ is neither even nor odd.

(e) $\square$ The function $f(x) = (x+3)^2 - 7$ is one-to-one on the interval $[-9, 0]$.

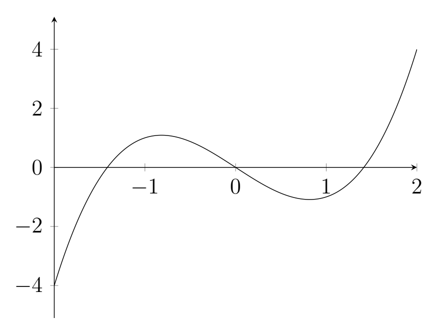
(f) ☐ Every positive decreasing sequence is bounded.

(g) ☐ Every positive increasing sequence is bounded.

(h) ☐ A function never has more than one y-intercept.

(i) ☐ If a function only has one x-intercept, it is one-to-one.

(j) ☐ The following graph has relative minima at $y = -2$ and $y = -4$.



2. (**20 points**) Let $f(x) = \dfrac{1}{\sqrt{x-2}}$, $g(x) = x^2$, $h(x) = x - 2$.

(a) Find the domain and the range of f and g.

(b) Write the equation of the function $g \circ f$ and find its domain and range.

(c) Write the equation of hf, and find its domain and range.

(d) Find the equation of f^{-1}, and specify its domain and range.

3. (**15 points**) Samantha would like to create a model for how long it takes her to swim some number of miles. She knows that it takes her 4 hours to swim 3 miles, 6 hours to swim 4 miles, and 8 hours to swim 6 miles. She knows the model should look like a line.

   (a) Recall that the Lagrange interpolation formula for two points (x_1, y_1), (x_2, y_2) is

   $$f(x) = y_1 \frac{x - x_2}{x_1 - x_2} + y_2 \frac{x - x_1}{x_2 - x_1}.$$

   From the statement of the problem, we have three points

   $$p_1 = (3, 4), \qquad p_2 = (4, 6), \qquad p_3(6, 8).$$

   Compute the line that goes through p_1 and p_2. Compute the line that goes through p_1 and p_3. Compute the line that goes through p_2 and p_3. Show your work.

   (b) Sketch all three lines and label them.

   (c) Which line makes the most sense as a model? Explain why.

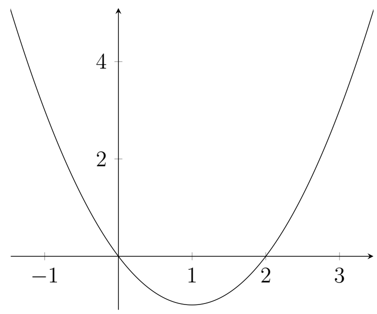
4. (**12 points**) Let $f(x)$ be a one-to-one function with domain $[-4, 5]$ and range $[-2, 4]$.

   (a) Write down the domain and range of
   $$3f(-x) + 2.$$
   Show your work.

   (b) Write down the domain and range of
   $$2f^{-1}(x + 1) - 3.$$
   Show your work.

5. (**12 points**) Consider the following classes of functions: **linear**, **power**, **polynomial**, **rational**. For each of the following graphs, write down which classes (if any) it belongs to, and which ones (if any) it doesn't belong to. Note that all four classes should be written down for each graph.

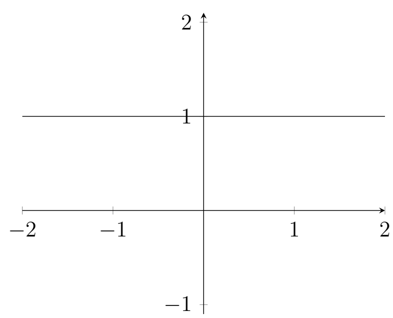
(a)



Belongs to:

Doesn't belong to:

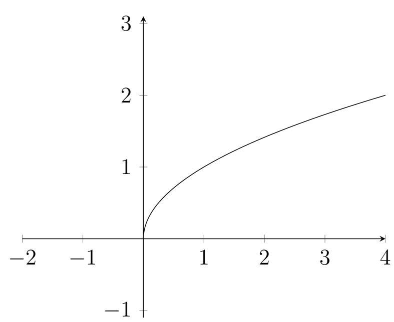
(b)



Belongs to:

Doesn't belong to:

(c)



Belongs to:

Doesn't belong to:

6. (**10 points**) Show all your work for the following problems.

   (a) Solve for x in the equation
   $$16^{(2^x)} = 2^{(4^x)}.$$

   (b) Solve for x in the equation
   $$\log_3(x) + \log_3(x+1) = \log_3(2).$$

7. (**7 points**) Let $f(x) = 3x^2 - 5x + 2$.

   (a) Find the average rate of change of f on the interval $[0, 1]$.

   (b) Find the average rate of change of f on the interval $[1, 2]$.