MATH 1 Homework 8

Assigned November 2nd, due November 9th

1. Let $xy - 4 = 4y^2$.

   (a) Use implicit differentiation to find $\frac{dy}{dx}$.

   (b) For the following points, does there exist a tangent line at that point? If so, find the equation of the tangent line.

      i. $(17, 4)$
      ii. $(10, 2)$
      iii. $(3, 3)$

2. Let $x^4 - 3y^2 = 2xy$. Use implicit differentiation to solve for the following:

   (a) $\frac{dy}{dx}$
   (b) $\frac{dx}{dy}$

3. Use implicit differentiation to show that $\frac{d}{dx}\operatorname{arcsec}(x) = \frac{1}{x\sqrt{x^2-1}}$. (*Hint:* $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$. *You do not need to show this.*)

4. Let $y = x^x$. Using natural logarithms and implicit differentiation, find $\frac{dy}{dx}$ in terms of x. (*Hint: start by taking the natural logarithm of both sides.*)

5. (a) Explain how we know that $3x^4 - 8x^3 + 2 = 0$ has a root in the interval $[2, 3]$.

   (b) Starting with $x_0 = 3$, do 3 iterations of Newton's method to approximate the root to three decimal places. Use a calculator, but write down the formulas for each iteration. Show all your work.