MATH 1 Homework 7 Solutions: total 53 points

Assigned October 26th, due November 2nd

1. (total 17 pts)

   (a) $\dfrac{d}{dx}(x^2\sin(x)) = \dfrac{d(x^2)}{dx}\sin(x) + x^2\dfrac{d}{dx}\sin(x) = 2x\sin(x) + x^2\cos(x)$ by the product rule

   (b) $\dfrac{d}{dx}\left(\dfrac{x^2+5x+3}{3x-2}\right) = \dfrac{(x^2+5x+3)'(3x-2)-(x^2+5x+3)(3x-2)'}{(3x-2)^2} =$
   $\dfrac{(2x+5)(3x-2)-3(x^2+5x+3)}{(3x-2)^2}$

   (c) $\dfrac{d}{dx}\left(\dfrac{3\cos(x)}{x}\right) = \dfrac{(3\cos(x))'x - 3\cos(x)(x)'}{x^2} = \dfrac{-3x\sin(x)-3\cos(x)}{x^2}$

2. $\dfrac{d}{dx}(\sec(x)) = \dfrac{d}{dx}\left(\dfrac{1}{\cos(x)}\right) = \dfrac{(1)'\cos(x) - 1\cdot(\cos(x))'}{cos^2(x)} = \dfrac{0-(-\sin(x))}{\cos^2(x)} = \dfrac{\sin(x)}{\cos^2(x)} =$
   $\dfrac{1}{\cos(x)}\dfrac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$

3. We need to take the derivative $n+1$ times: after the first derivative, we get an $(n-1)$-degree polynomial, after the second: an $(n-2)$-degree one, after the third an $(n-3)$-degree one etc. and we get the constant zero function when we have to take the derivative of a constant, or a 0-degree polynomial. We will get a 0-degree polynomial after taking n-derivatives. In total, this will make $n+1$ derivatives.

   Example: let $P(x) = x^3 + x^2 + x + 1$ a 3-degree polynomial, then $P'(x) = 3x^2 + 2x + 1$, $P''(x) = 6x + 2$, $P^{(3)}(x) = 6$, and finally $P^{(4)}(x) = 0$.

4. (a) The derivative of $h(k(x))$ is $h'(k(x))\cdot k'(x)$.

   (b) Let's think of $h(k(x))$ as the inside function and $g(x)$ as the outer function. Then the chain rule says $[g(h(k(x)))]' = g'(h(k(x)))\cdot[h(k(x))]'$. We know from part a that $[h(k(x))]' = h'(k(x))\cdot k'(x)$, so $[g(h(k(x)))]' = g'(h(k(x)))\cdot h'(k(x))\cdot k'(x)$.

   (c) We know from part b that $f'(x) = g'(h(k(x)))\cdot h'(k(x))\cdot k'(x)$, so $f'(2) = g'(h(k(2)))\cdot h'(k(2))\cdot k'(2)$. Now we just need to plug in.

$$\begin{aligned} f'(2) &= g'(h(k(2)))\cdot h'(k(2))\cdot k'(2) \\ &= g'(h(3))\cdot h'(3)\cdot 6 \\ &= g'(4)\cdot 5\cdot 6 \\ &= 2\cdot 5\cdot 6 \\ &= 60 \end{aligned}$$

5. (a) Let $g(x) = \cos(x)$ and let $h(x) = \sqrt{x}$, then $f(x) = g(h(x))$. Since $g'(x) = -\sin(x)$ and $h'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$, then by the chain rule, we have $f'(x) = -\sin(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$.

(b) Let $f(x) = e^x$ and let $h(x) = x\sin(x)$, then $g(x) = f(h(x))$. We need to use the product rule to find $h'(x)$, so we get $h'(x) = \sin(x) + x\cos(x)$. Then by the chain rule, we have $g'(x) = e^{x\sin(x)} \cdot (\sin(x) + x\cos(x))$.

(c) Let $f(x) = \sin(x)$, $g(x) = e^x$, and $k(x) = 2x - 5$, then $h(x) = f(g(k(x)))$. We can use what we did in the previous question to find that $h'(x) = \cos(e^{2x-5}) \cdot (e^{2x-5}) \cdot 2$.