MATH 1 Homework 7

Assigned October 26th, due November 2nd

1. Find the following derivatives:

   (a) $\dfrac{d}{dx}(x^2 \sin(x))$

   (b) $\dfrac{d}{dx}\left(\dfrac{x^2+5x+3}{3x-2}\right)$

   (c) $\dfrac{d}{dx}\left(\dfrac{3\cos(x)}{x}\right)$

2. Show that
$$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x).$$

3. Let $P(x)$ be a polynomial of degree n. How many derivatives of $P(x)$ do you need to take to get the constant 0 function? Explain your answer. *Hint: Try a couple of specific examples and see if you can come up with a hypothesis.*

4. Suppose $f(x) = g(h(k(x)))$ for some functions g, h, k. Denote the derivative of $k(x)$ as $k'(x)$, the derivative of $h(x)$ as $h'(x)$, and the derivative of $g(x)$ as $g'(x)$.

   (a) Using the chain rule, what is the formula for the derivative of $h(k(x))$?

   (b) Using the chain rule, what is the formula for the derivative of $g(h(k(x)))$?

   (c) Suppose $k(2) = 3$, $k'(2) = 6$, $h(3) = 4$, $h'(3) = 5$, $g(4) = 7$, and $g'(4) = 2$. Then what is $f'(2)$?

5. (a) Let $f(x) = \cos(\sqrt{x})$. Write $f(x)$ as the composition of two nonidentity functions, and then use the chain rule to find $f'(x)$.

   (b) Let $g(x) = e^{\sin(x)}$. Write $g(x)$ as the composition of two nonidentity functions, and then use the chain rule to find $g'(x)$.

   (c) Let $h(x) = \sin(e^{2x-5})$. Write $h(x)$ as the composition of three nonidentity functions, and then use the chain rule to find $h'(x)$.