MATH 1 Homework 5

Assigned October 12th, due October 19th

1. Let $f(x) = \frac{1}{(x-1)^4}$. Answer the following questions, and justify your answers.

   (a) How close to 1 does x have to be so that $\frac{1}{(x-1)^4} > 10000$?

   (b) How close to 1 does x have to be so that $\frac{1}{(x-1)^4} > 160000$?

   (c) Find $\lim_{x\to 1} f(x)$.

2. Find functions f and g such that $\lim_{x\to 1} g(x) = 3$ but $\lim_{x\to 1} f(g(x)) \neq f(3)$. (Either draw the graphs of the functions, or give their equations).

3. Write down an equation for a function f such that f has horizontal asymptote $y = 1$, vertical asymptote $x = 3$, and $\lim_{x\to 5} f(x) = 3$.

4. This exercise will give you some practice to explore the $\epsilon - \delta$ concepts of the limit. Each of the following functions is a polynomial, so the limit $\lim_{x\to a} f(x) = f(a)$. Answer each of the following questions, and justify you answers.

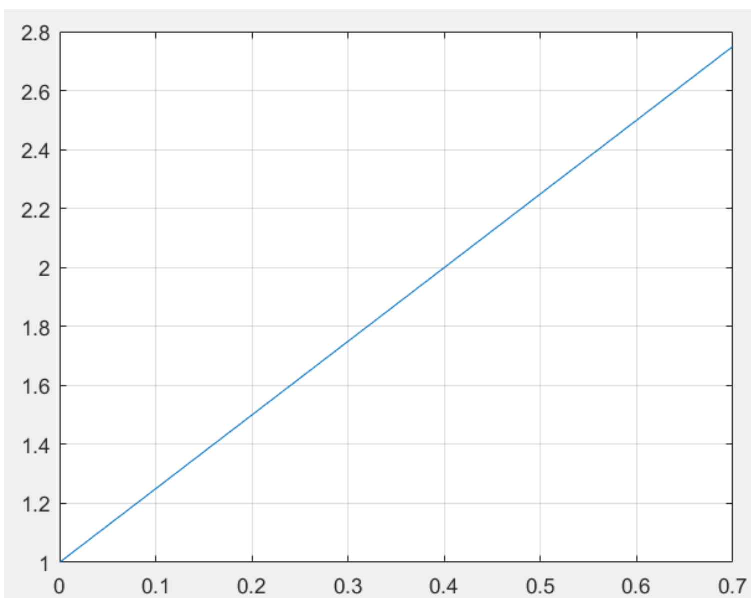

Figure 1. Graph of a function f

   (a) In Figure 1 we have the graph of a function f when $0 \leq x \leq 0.7$. As we see, $\lim_{x\to 0.4} f(x) = 2$. Give a value for δ so that when x is δ-close to 0.4 (i.e. $|x-0.4| < \delta$), then $|f(x)-2| < 0.2$.

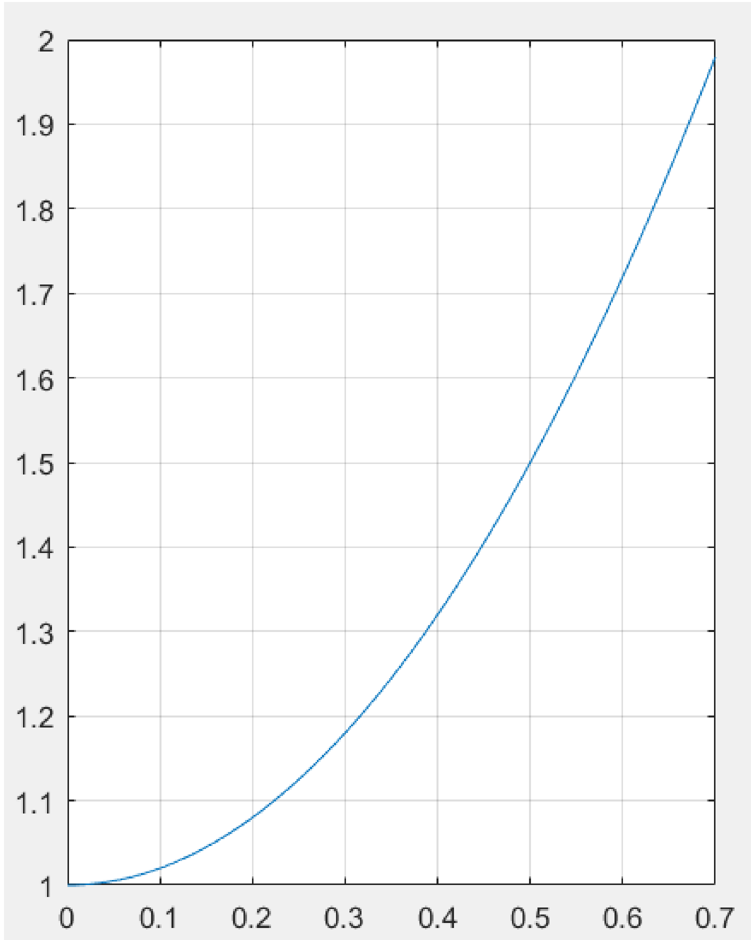

Figure 2. Graph of a function g

(b) In Figure 2, we have the graph of a function g when $0 \leq x \leq 0.7$. We see $\lim_{x \to 0.5} g(x) = 1.5$. Give a value for δ so that when $|x - 0.5| < \delta$, then $|g(x) - 1.5| < 0.2$. Give a value for δ so that when $|x - 0.5| < \delta$, then $|g(x) - 1.5| < 0.1$.

5. Give three examples of different types of functions that are discontinuous at infinitely many points. *Hint: we have talked about some such functions at the beginning of the term, while classifying functions.*

6. Are the following functions continuous at the given points? Why or why not?

   (a) $f(x) = \cos\left(\frac{x+3}{x^2-2}\right)$ at $x = 5$.

   (b) $g(x) = \tan\left(\frac{x - \frac{\pi}{2}}{x - \pi + 1}\right)$ at $x = \pi$.

   (c) $h(x) = \ln(x^2 - 3)$ at $x = 2$.

   (d) $k(x) = 2^{\log_3(\sqrt{x})}$ at $x = 17$.

7. Use the Squeeze Theorem to evaluate the following limits. Show your work.

   (a) $\lim_{x \to 0} x^2 \arctan\left(\frac{1}{x}\right)$.

   (b) $\lim_{x \to 0} x e^{\sin\left(\frac{1}{x}\right)}$.

   (c) $\lim_{x \to -3} (x+3) \cos\left(\frac{1}{x+3}\right)$.

8. Evaluate the following limits without graphing or calculating points.

   (a) $\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$.

   (b) $\lim_{x \to 2} \frac{\sqrt{4x+1} - 3}{x-2}$.