MATH 1 Homework 4

Assigned October 5th, due October 12th

1. Do the following sequences converge? If so, what to?
   (a) $\{\frac{1}{n^2}\}_{n=1}^{\infty}$
   (b) $\{\cos(n\pi)\}_{n=1}^{\infty}$
   (c) $\{n^3\}_{n=1}^{\infty}$
   (d) $\{\frac{2n-2}{n}\}_{n=1}^{\infty}$
   (e) $\{\sin^2(n)+\cos^2(n)\}_{n=1}^{\infty}$

2. For the following scenarios, give an example or explain why it can't happen. Can a sequence converge if it is...
   (a) monotone and bounded?
   (b) monotone and unbounded?
   (c) bounded but not monotone?

3. Let $\{a_n\} = \{0, x^2, 0, x^4, 0, x^6, \dots\}$. If $x = \frac{1}{3}$, the sequence converges. If $x = 2$, the sequence does not converge. Find all values of x for which the sequence converges. Explain why the sequence converges for the values of x that you found, and explain why it does not converge for other values of x.

4. Let $\{a_n\}_{n=1}^{\infty} = \{3^n\}_{n=1}^{\infty}$.
   (a) Find a sequence $\{b_n\}_{n=1}^{\infty}$ such that the product sequence $\{a_n b_n\}_{n=1}^{\infty}$ converges to 0. Explain why your answer converges to 0.
   (b) Find a sequence $\{c_n\}_{n=1}^{\infty}$ such that the quotient sequence $\{\frac{a_n}{c_n}\}_{n=1}^{\infty}$ converges to 1. Explain why your answer converges to 1.
   (c) Find a sequence $\{d_n\}_{n=1}^{\infty}$ such that the quotient sequence $\{\frac{a_n}{d_n}\}_{n=1}^{\infty}$ does not converge. Explain why it doesn't converge.

5. Either plot the graphs of each of the following functions or approximate them numerically in order to guess the following limits. Show your work.
   (a) $\lim_{x\to 1} \dfrac{x^2+3x-4}{x-1}$
   (b) $\lim_{x\to 1} [\ln(-x^2+4x-3) - \ln(x-1)]$
   (c) $\lim_{x\to 0} \dfrac{\sin(x)\tan(x)}{x^2}$

6. What are the asymptotes of the following functions? For parts (a) and (b), sketch the function.
   (a) $\dfrac{x^2+1}{x^2}$
   (b) $\dfrac{5x}{3x+3}$
   (c) $\ln(x^2+x-6)$