MATH 1 Homework 1

Assigned September 14th, due September 21st

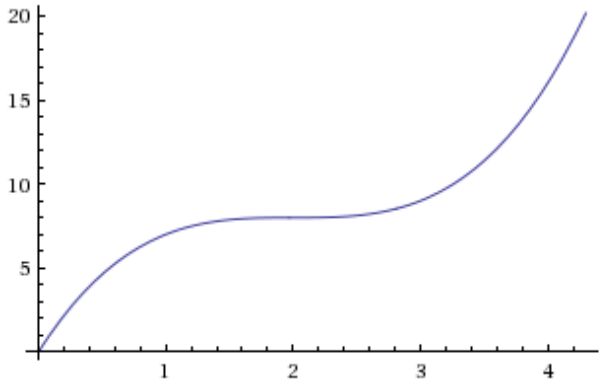
1. Let
$$f(x) = -(x-1)(x+1), \qquad g(x) = 2x, \qquad h(x) = \sqrt{x}.$$
Write down the equation for each of the following functions, and find their domain and range:
$$f,\ g,\ h,\ g+h,\ \frac{1}{g},\ f \circ h,\ h \circ f.$$

2. Write down the equation for each of the following functions and sketch their graphs:

   (a) the area of a square as a function of the length of a side;
   (b) the length of the side of a square as a function of the area of the square.

   Identify three points that lie on each graph, and write them down in the form $(x, f(x))$.

3.



Let f be defined on all the real numbers. In the figure above, we have the partial graph for f when $x \geq 0$.

   (a) Assuming that f is odd, sketch the rest of the graph;
   (b) Assuming that f is even, sketch the rest of the graph;
   (c) For an arbitrary function h, can h be both increasing and even? How about increasing and odd? Explain in your own words why or why not.

4. (a) Consider the following sequences

$$\{a_n\} = \{n\}_{n=1}^{\infty}$$

$$\{b_n\} = \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

$$\{c_n\} = \left\{\frac{a_n}{b_n}\right\}_{n=1}^{\infty}$$

$$\{d_n\} = \{(-1)^n\}_{n=1}^{\infty}$$

(In the sequence $\{c_n\}$, division is done term by term).

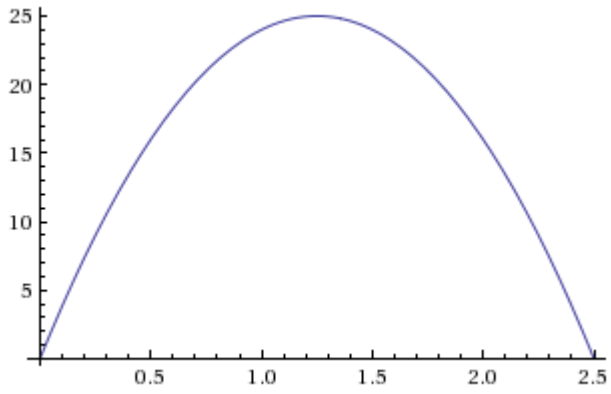
For each of the 4 sequences, answer the following questions: Is it increasing, decreasing, or neither? Is it bounded? If it is bounded, give a bound for the sequence.

(b) Consider a positive sequence $\{l_n\} = \left\{\frac{p_n}{q_n}\right\}_{n=1}^{\infty}$, where $\{p_n\}$ and $\{q_n\}$ are two other positive sequences (*a positive sequence is a sequence where each term is positive*).

If both $\{l_n\}$ and $\{p_n\}$ are increasing, can $\{q_n\}$ be increasing? Can it be decreasing? If so, give an example. If not, explain why not.

If $\{p_n\}$ is increasing and $\{l_n\}$ is decreasing, can $\{q_n\}$ be increasing? Can it be decreasing? If so, give an example. If not, explain why not.

5. If a rock is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $y = 40t - 16t^2$ (see the graph below).



Find the average velocity over the given time intervals:

(a) $[2, 2.5]$

(b) $[2, 2.1]$

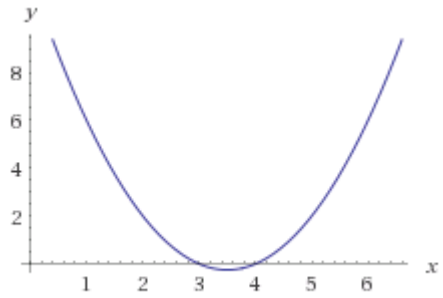
(c) $[2, 2.05]$

(d) $[2, 2.01]$

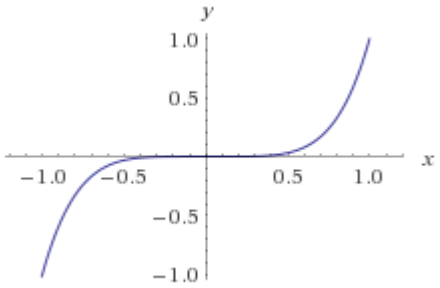
What do you notice about your answers?

6. Recall that we talked about the following types of functions: constant, linear, power, polynomial, rational, and algebraic. For each of the following graphs, write down what types of function it is NOT. Write as many as you can, and justify your answers.

(a)



(b)



(c)