

# Limit Laws

Let  $f(x), g(x)$ , and  $h(x)$  be functions,  $a$  a number, and  $c$  a constant. If

$$\lim_{x \rightarrow a} f(x), \quad \lim_{x \rightarrow a} g(x) \quad \text{and} \quad \lim_{x \rightarrow a} h(x)$$

exist, then the following hold

1.

$$\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

2.

$$\lim_{x \rightarrow a} (f(x) - g(x)) = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

3.

$$\lim_{x \rightarrow a} (f(x)g(x)) = \left( \lim_{x \rightarrow a} f(x) \right) \lim_{x \rightarrow a} g(x)$$

4.

$$\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$$

5.

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad \text{if} \quad \lim_{x \rightarrow a} g(x) \neq 0$$

6.

$$\lim_{x \rightarrow a} f(x)^n = \left( \lim_{x \rightarrow a} f(x) \right)^n \quad \text{where } n \text{ is a positive integer}$$

7.

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} \quad \begin{array}{l} \text{where } n \text{ is a positive integer} \\ \text{(if } n \text{ is even, the limit must be positive)} \end{array}$$

8. If  $f(x) = g(x)$  when  $x \neq a$ , then

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$$

9. If  $f(x) \leq g(x) \leq h(x)$  when  $x$  is near  $a$  (except possibly at  $a$ ) and

$$\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$$

then

$$\lim_{x \rightarrow a} g(x) = L$$

# Limits of common functions

1.

$$\lim_{x \rightarrow a} c = c \quad \text{for any constant } c$$

2.

$$\lim_{x \rightarrow a} x = a$$

3. For any rational function  $f(x)$ , if  $a$  is in the domain of  $f$ , then

$$\lim_{x \rightarrow a} f(x) = f(a)$$

4.

$$\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a} \quad \begin{array}{l} \text{where } n \text{ is a positive integer} \\ \text{(if } n \text{ is even, then } a \text{ must be positive)} \end{array}$$

5.

$$\lim_{x \rightarrow a} \cos(x) = \cos(a)$$

6.

$$\lim_{x \rightarrow a} \sin(x) = \sin(a)$$

7.

$$\lim_{x \rightarrow a} b^x = b^a \quad \text{for any } b > 0$$

8.

$$\lim_{x \rightarrow a} \log_b(x) = \log_b(a) \quad \text{for any } b > 0, b \neq 1, \text{ and } a > 0$$

9.

$$\lim_{x \rightarrow 0^+} \log_b(x) = -\infty \quad \text{for any } b > 0, b \neq 1$$

10.

$$\lim_{x \rightarrow a} \tan(x) = \tan(a) \quad \text{for any } a \neq \left(\frac{\pi}{2} + \pi k\right), k \text{ an integer.}$$

11.

$$\lim_{x \rightarrow a^+} \tan(x) = -\infty \quad \text{for any } a = \left(\frac{\pi}{2} + \pi k\right), k \text{ an integer.}$$

12.

$$\lim_{x \rightarrow a^-} \tan(x) = \infty \quad \text{for any } a = \left(\frac{\pi}{2} + \pi k\right), k \text{ an integer.}$$