Math 1 Lecture 9 Friday 09-30-16

Michael Musty
Dartmouth College

09-30-16

Reminders/Announcements

- Exams will be back in the HW boxes later today
- WebWork due Monday
- Written HW due Wednesday
- Quiz Monday

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- We have a lot of stuff going on!

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The ratio $\frac{c}{d}$ does not depend on the circle.
This constant ratio is known as π.

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}.$$

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We will also just write π instead of $\pi\,\text{rad}$.

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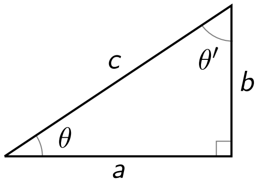
$$\theta' = \frac{\pi}{2} - \theta$$



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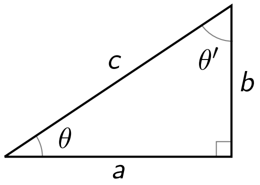

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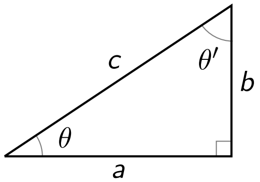

Remember, in Euclidean geometry, interior angles of triangles add up to π. Remember also that a, b, c satisfy a certain equation...

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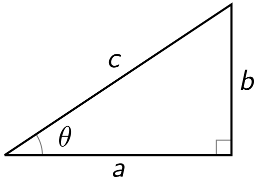

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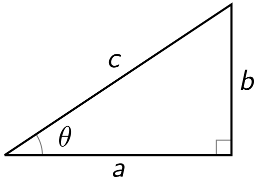

I mean, nobody can stop us from just making a definition!

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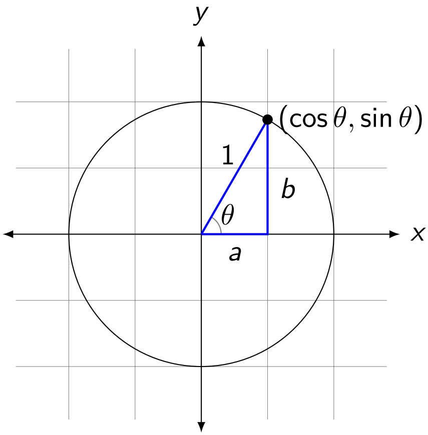

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In summary, we have defined 6 functions
$\sin\theta, \cos\theta, \tan\theta, \sec\theta, \csc\theta, \cot\theta$.

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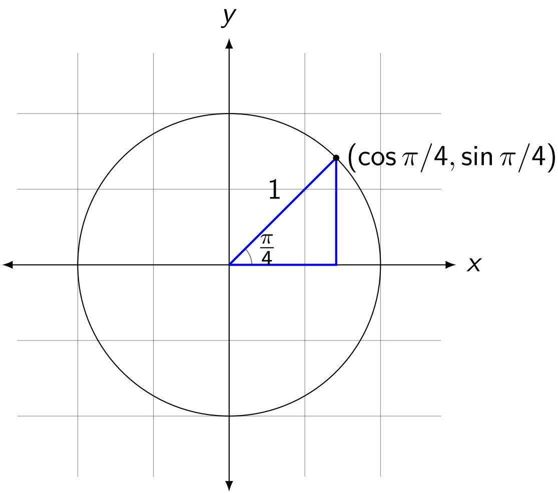
In summary, we have defined 6 functions $\sin\theta, \cos\theta, \tan\theta, \sec\theta, \csc\theta, \cot\theta$. We will mostly be concerned with the first 3, but good to know them all!

$\cos \pi/4$ and $\sin \pi/4$



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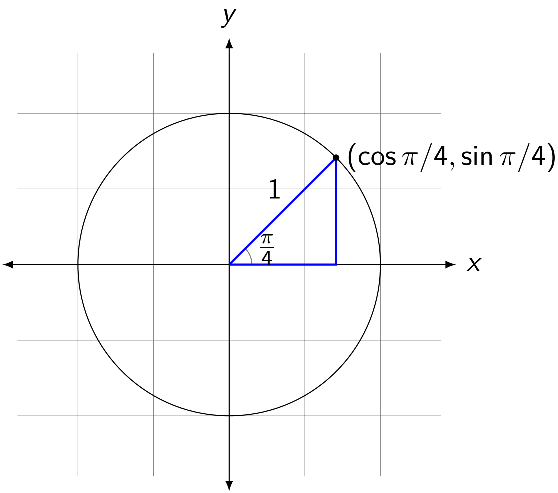
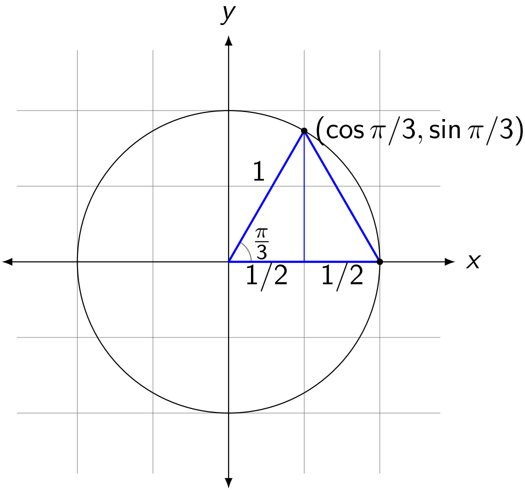


$$(\cos \pi/4, \sin \pi/4) = (\sqrt{2}/2, \sqrt{2}/2).$$

$\cos \pi/3$ and $\sin \pi/3$



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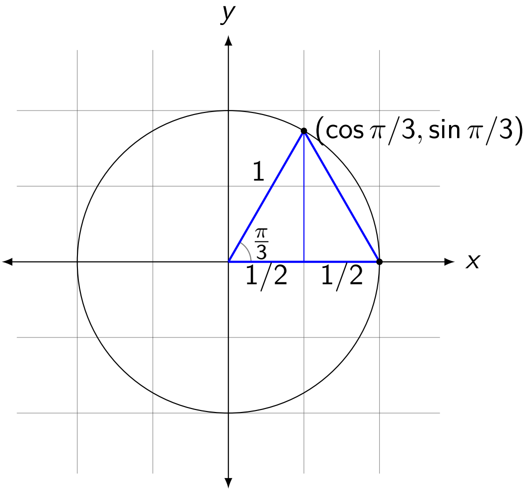
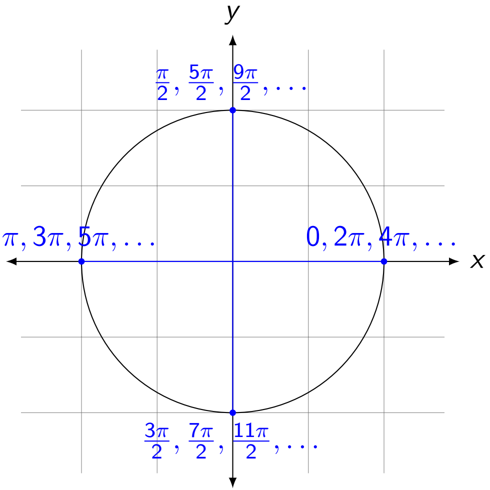


$$(\cos \pi/3, \sin \pi/3) = (1/2, \sqrt{3}/2).$$

Positive Multiples of $\frac{\pi}{2}$



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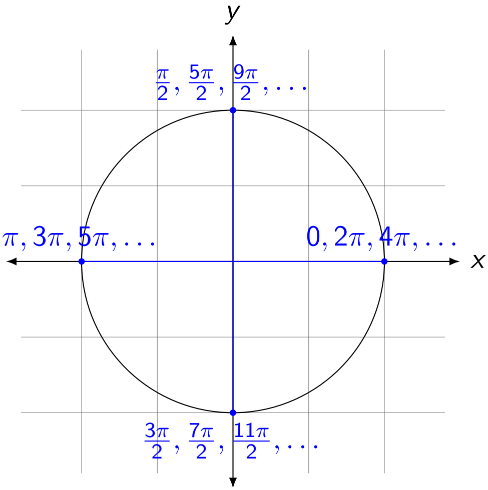

$$(\cos 7\pi/2, \sin 7\pi/2) =$$

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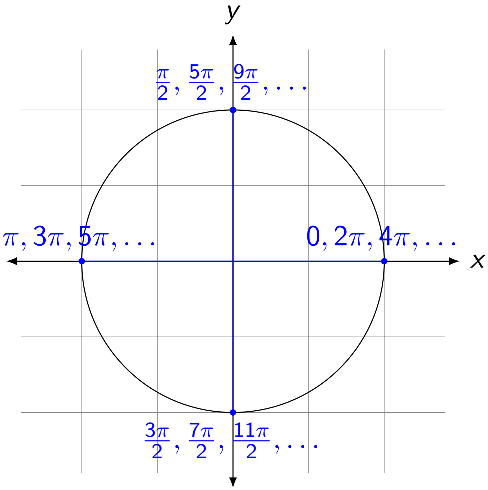

$$(\cos 7\pi/2, \sin 7\pi/2) = (0, -1)$$

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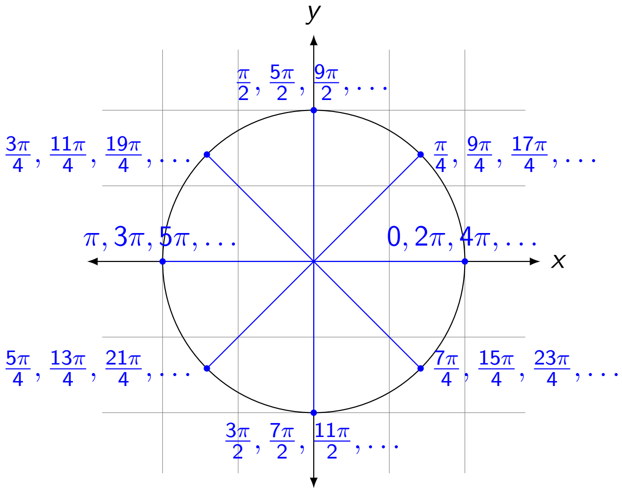

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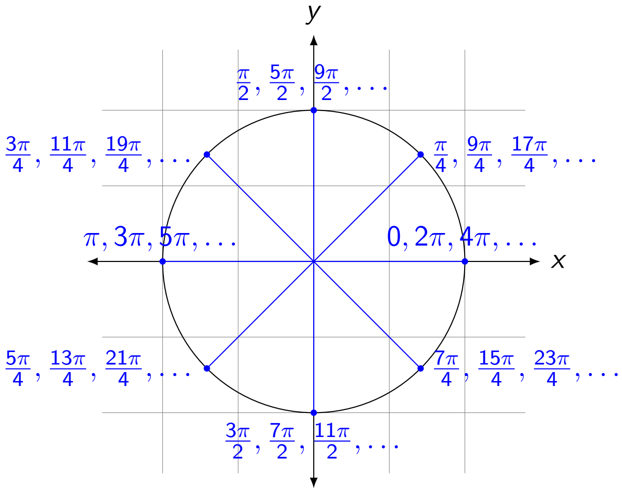

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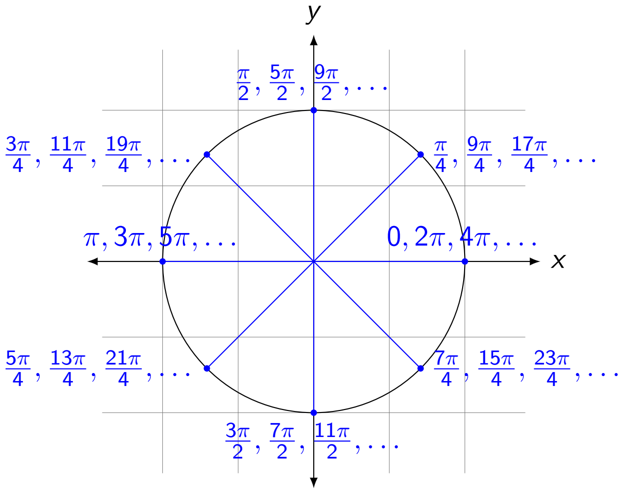

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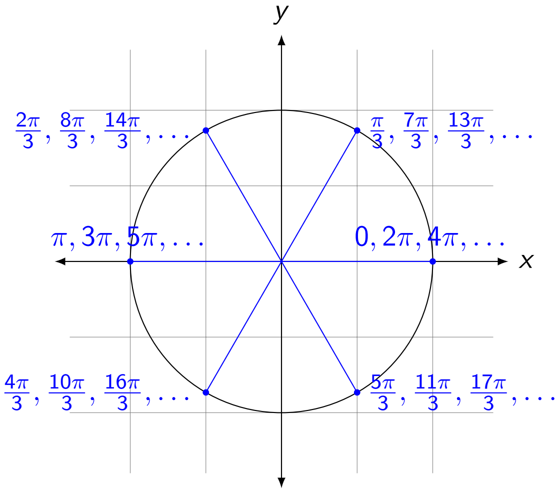

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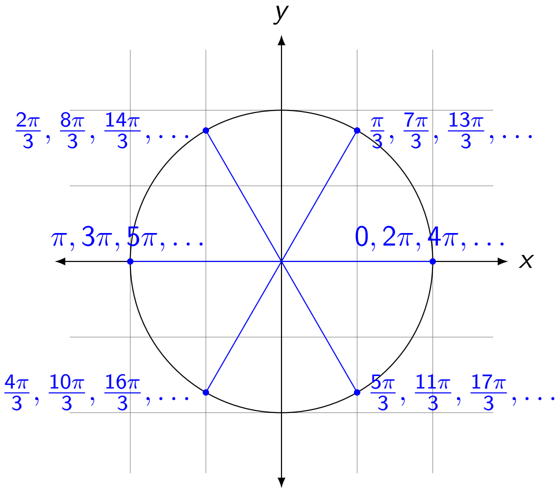

$$(\cos 14\pi/3, \sin 14\pi/3) =$$

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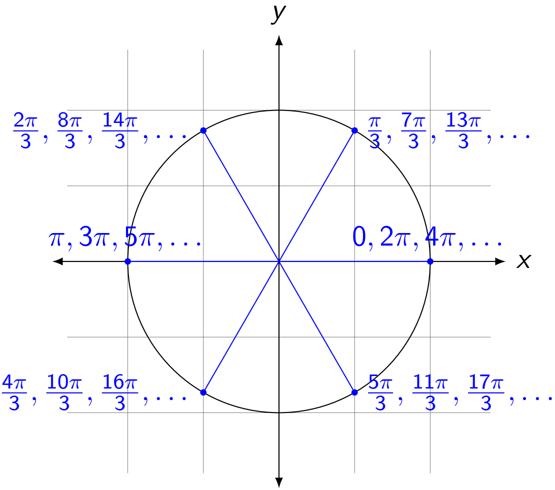
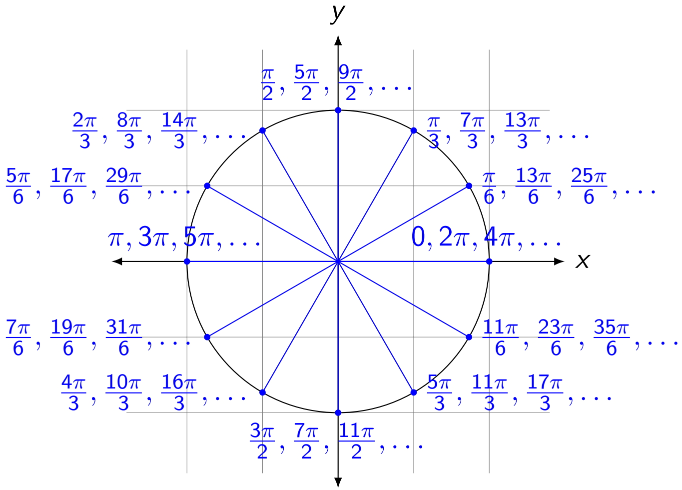


$$(\cos 14\pi/3, \sin 14\pi/3) = (-1/2, \sqrt{3}/2)$$

Positive Multiples of $\frac{\pi}{6}$



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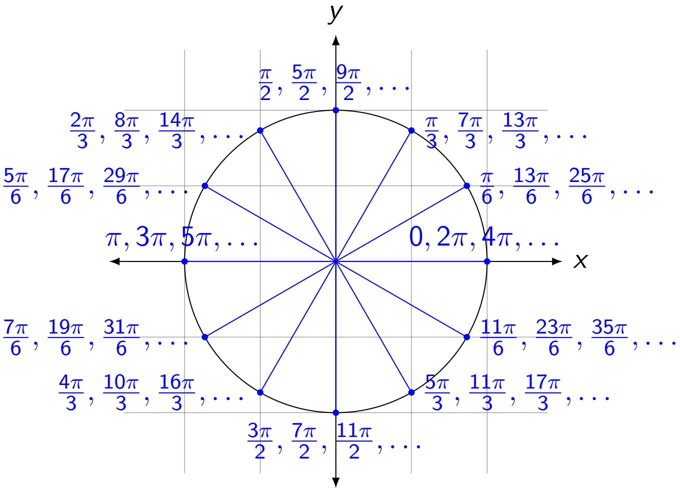

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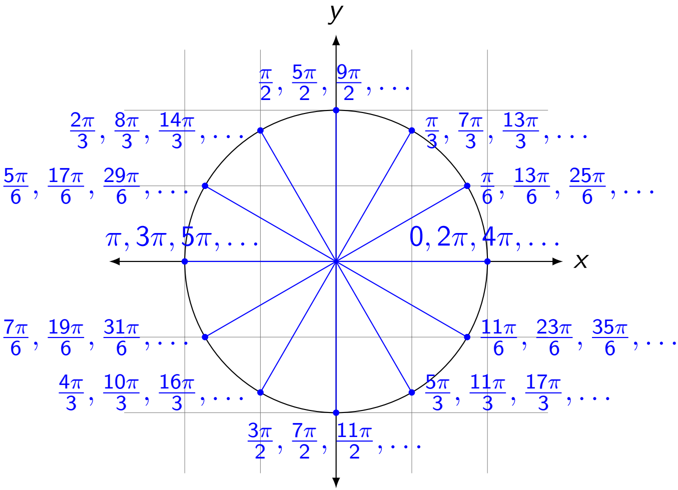

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For a brief summary of this information dump see our website:
`https://math.dartmouth.edu/~m1f16/MATH1Docs/TrigonometryReview.pdf`

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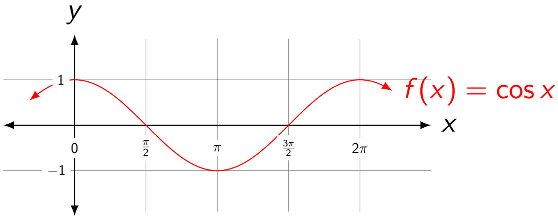
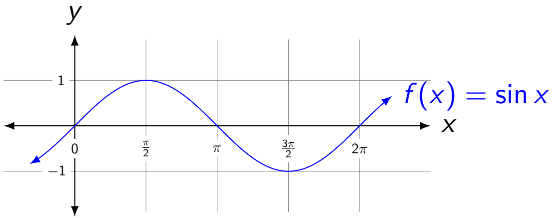
Now we can study the 6 **trigonometric functions** as functions $f : \mathbb{R} \to \mathbb{R}$.
We will commonly write these functions in the variable x instead of θ.

$f(x) = \sin x$ and $f(x) = \cos x$





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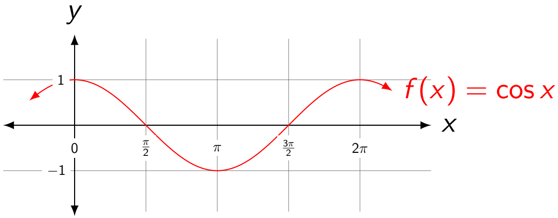

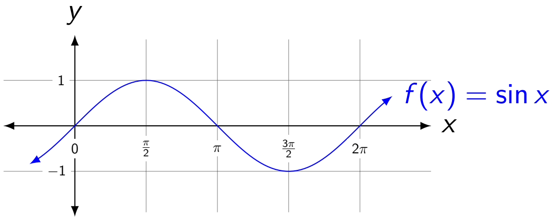

Domain?

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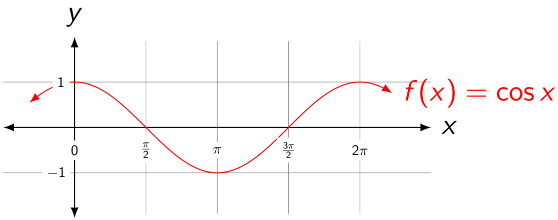

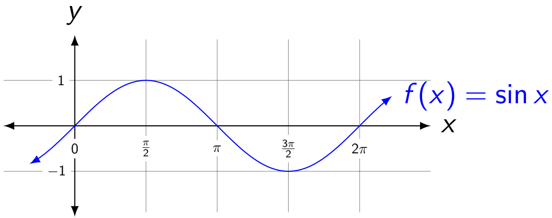

Domain? $(-\infty, \infty)$.

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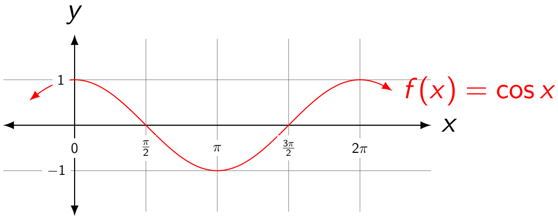
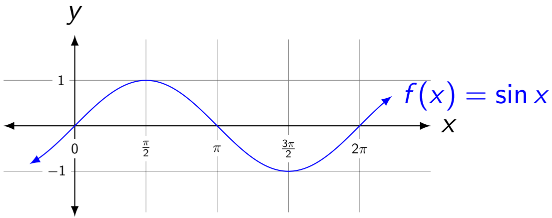
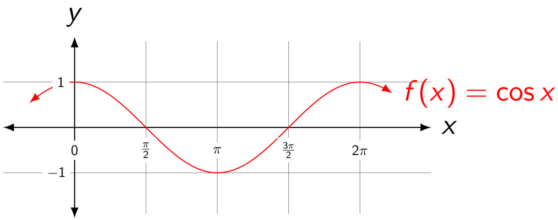
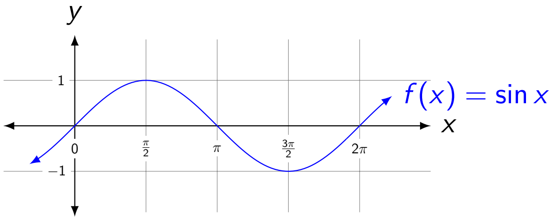




Domain? $(-\infty, \infty)$. Range?

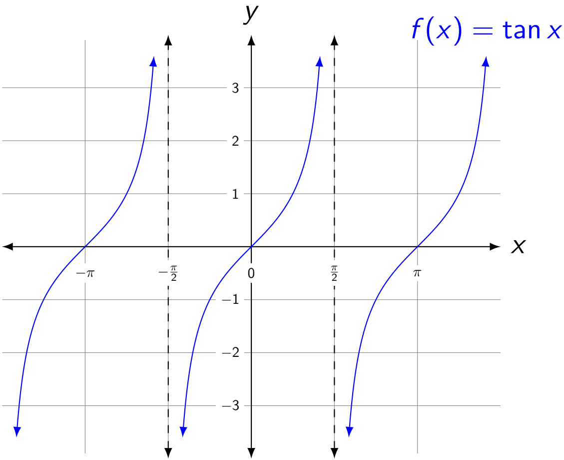
$f(x) = \sin x$ and $f(x) = \cos x$





Domain? $(-\infty, \infty)$. Range? $[-1, 1]$.

$f(x) = \tan x$



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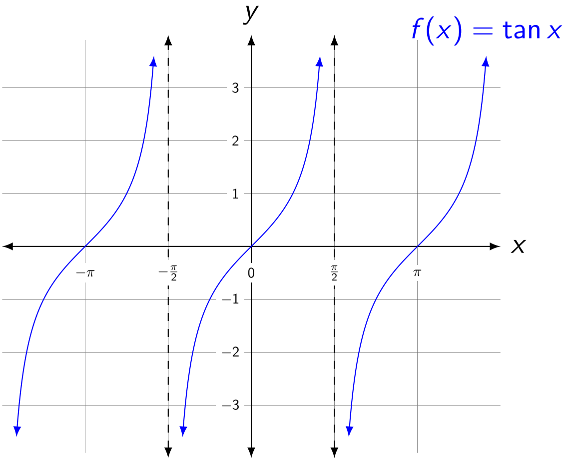
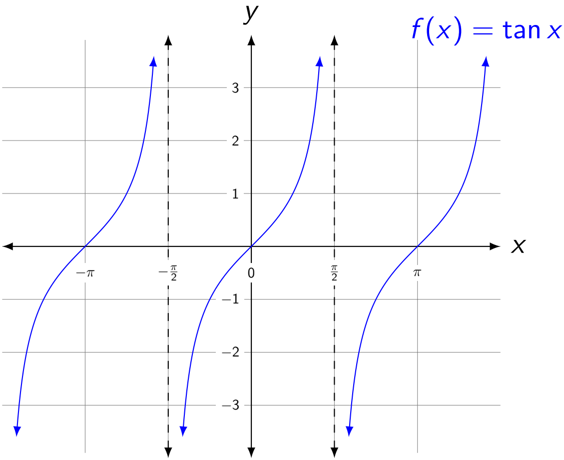


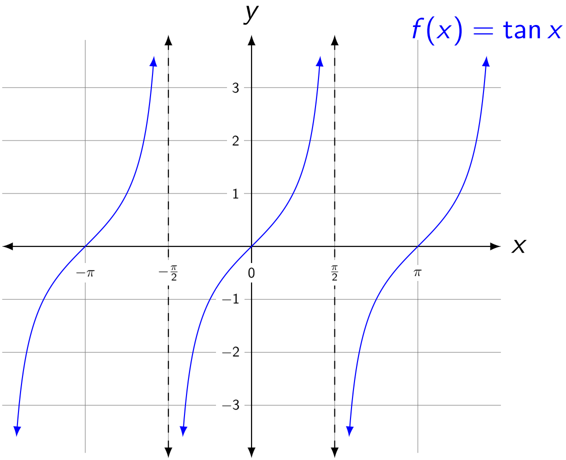
Domain?

$f(x) = \tan x$



Domain? Exclude the odd multiples of $\pi/2$.

$f(x) = \tan x$



Domain? Exclude the odd multiples of $\pi/2$. Why?

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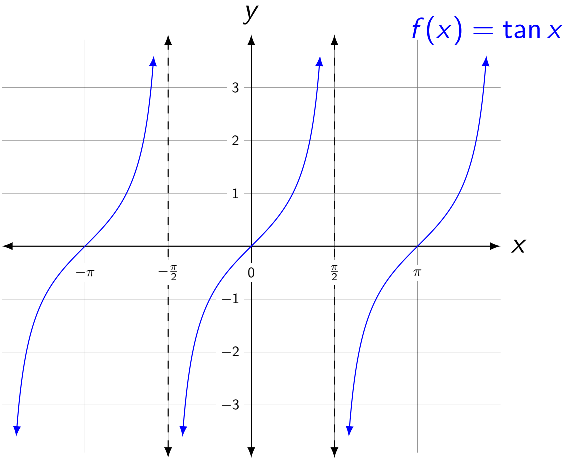
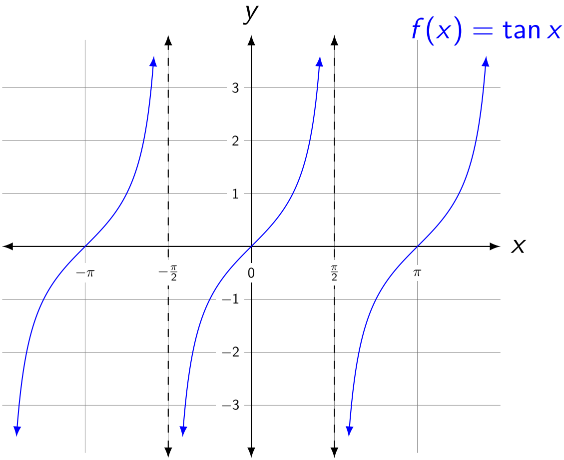


Domain? Exclude the odd multiples of $\pi/2$. Why? Because that is where $\cos x = 0$.

$f(x) = \tan x$



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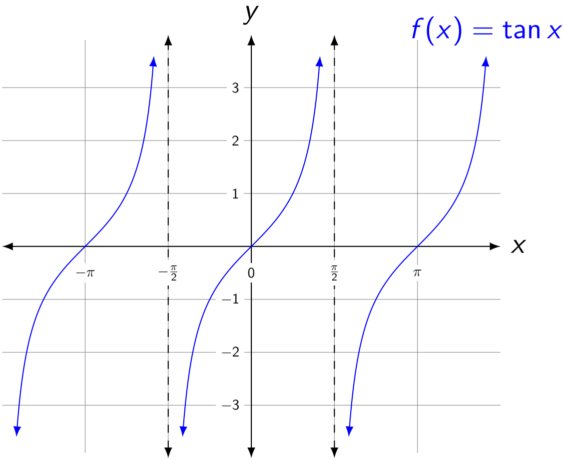
$f(x) = \tan x$



Domain? Exclude the odd multiples of $\pi/2$. Why? Because that is where $\cos x = 0$. Range? $(-\infty, \infty)$.

Period and Amplitude

An interesting property of the functions $\sin x$ and $\cos x$ is that they resemble waves.

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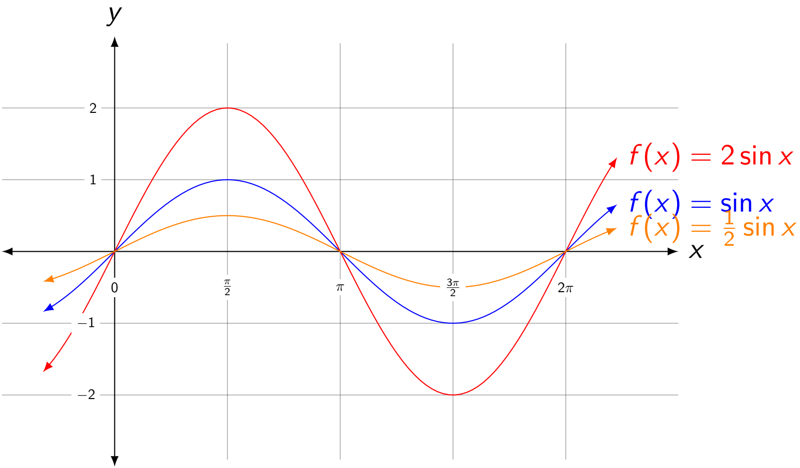
Uh, don't write something like that without showing an example!

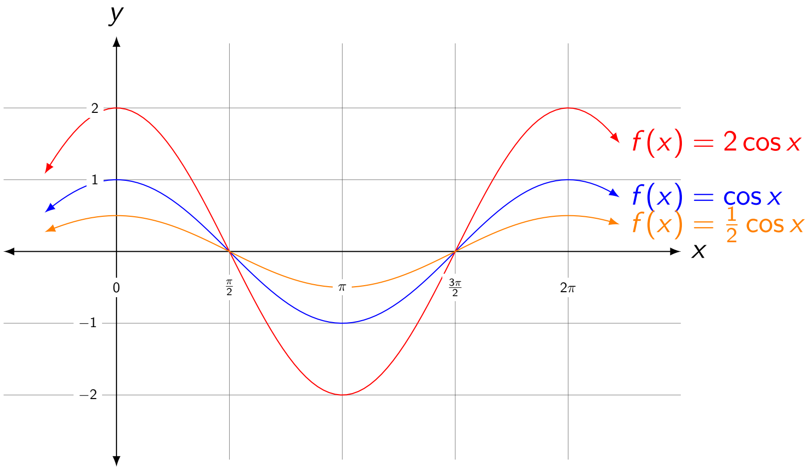
$f(x) = 2\sin x$ Amplitude 2

$f(x) = \sin x$ Amplitude 1

$f(x) = \frac{1}{2}\sin x$ Amplitude $\frac{1}{2}$



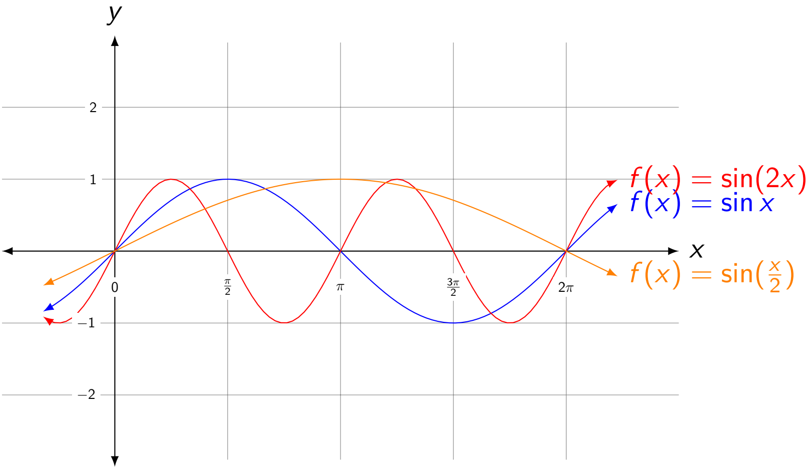
$f(x) = 2\cos x$	Amplitude 2
$f(x) = \cos x$	Amplitude 1
$f(x) = \frac{1}{2}\cos x$	Amplitude $\frac{1}{2}$



$f(x) = \sin(2x)$ Period π

$f(x) = \sin x$ Period 2π

$f(x) = \sin\left(\dfrac{x}{2}\right)$ Period 4π



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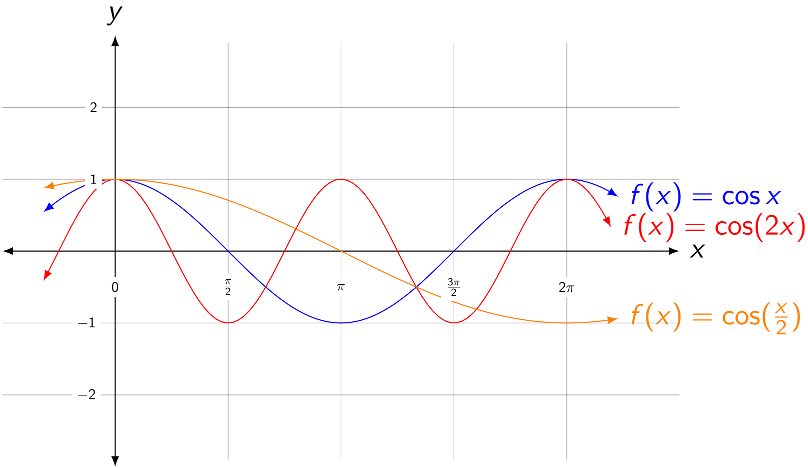

Period of $f(x) = \tan x$

The function $f(x) = \tan x$ does not have an amplitude, but it does have a period. What is it?

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Period of $f(x) = \tan x$

The function $f(x) = \tan x$ does not have an amplitude, but it does have a period. What is it? Remember the picture... the period is π.

$f(x) = a\sin(\omega x + t) + d$

Let a, t, d in $\mathbb{R}$ and $\omega > 0$. We would like to describe the function

$$f(x) = a\sin(\omega x + t) + d = a\sin\left(\omega\left(x + \frac{t}{\omega}\right)\right) + d$$

relative to the function $\sin x$.

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The period is $2\pi/\omega$.

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Horizontal shift by t/ω.

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Horizontal shift by t/ω.
Vertical shift by d.

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relative to the function $\sin x$.
The amplitude is a.
The period is $2\pi/\omega$.
Horizontal shift by t/ω.
Vertical shift by d.
Similar for $f(x) = a \cos(\omega x + t) + d$.

Please find the period and amplitude of the function $f(x) = 3\sin(7x)$.

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Solution:

The amplitude is 3 and the period is $2\pi/7$.

Please find the period and amplitude of the function $f(x) = 3\sin(7x - 1)$.

Please find the period and amplitude of the function
$f(x) = 3\sin(7x - 1)$.

Solution:

The amplitude is 3 and the period is $2\pi/7$.

Please find the period of the function $f(x) = 3\tan(7x)$.

Please find the period of the function $f(x) = 3\tan(7x)$.
Solution:
The period is $\pi/7$.

Please find the period of the function $f(x) = 3\tan(7x - 1)$.

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Solution:
The period is $\pi/7$.
What about the horizontal shift?

Please find the period of the function $f(x) = 3\tan(7x - 1)$.
Solution:
The period is $\pi/7$.
What about the horizontal shift? Right by $1/7$.

Are any of these functions one-to-one?

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Well. . . certainly not on their entire domain!

Are any of these functions one-to-one?

Well. . . certainly not on their entire domain! However, as we've seen before, we can consider functions on smaller domains where they are one-to-one and therefore have well-defined inverse functions.

$f(x) = \arcsin(x)$

To make $\sin x$ one-to-one we restrict to the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Then define the **arcsine** function by the following.

$$\arcsin x = y \iff \sin y = x$$

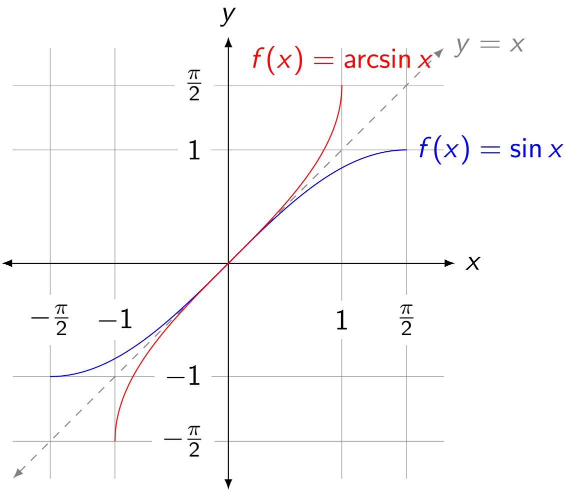
- The domain of $\arcsin x$ is $[-1, 1]$.
- The range of $\arcsin x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- For all x in $[-1, 1]$ we have that

$$\sin(\arcsin x) = x$$

- For all x in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ we have that

$$\arcsin(\sin x) = x$$

$f(x) = \arcsin x$



y
$y = x$
$f(x) = \arcsin x$
$\frac{\pi}{2}$
1
$f(x) = \sin x$
x
$-\frac{\pi}{2}$
-1
1
$\frac{\pi}{2}$
-1
$-\frac{\pi}{2}$

$f(x) = \arccos(x)$

To make $\cos x$ one-to-one we restrict to the domain $[0, \pi]$. Then define the **arccosine** function by the following.

$$\arccos x = y \iff \cos y = x$$

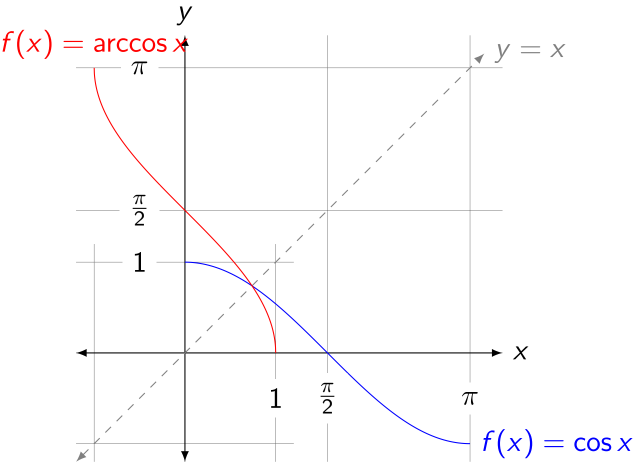
- The domain of $\arccos x$ is $[-1, 1]$.
- The range of $\arccos x$ is $[0, \pi]$.
- For all x in $[-1, 1]$ we have that

$$\cos(\arccos x) = x$$

- For all x in $[0, \pi]$ we have that

$$\arccos(\cos x) = x$$

$f(x) = \arccos x$



$f(x) = \arctan(x)$

To make $\tan x$ one-to-one we restrict to the domain $(-\frac{\pi}{2}, \frac{\pi}{2})$.
Then define the **arctangent** function by the following.

$$\arctan x = y \iff \tan y = x$$

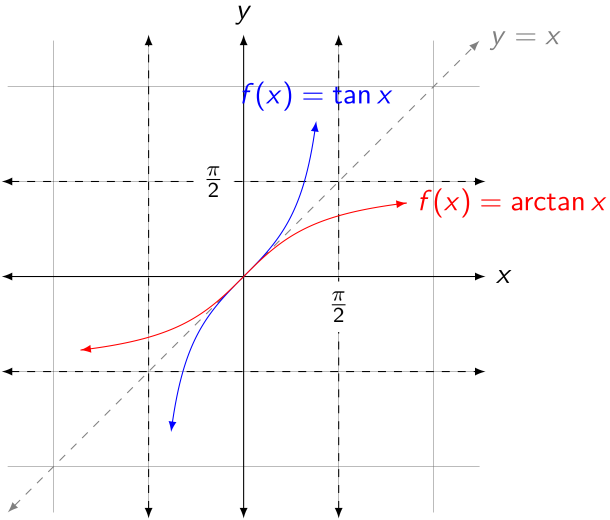
- The domain of $\arctan x$ is $\mathbb{R}$.
- The range of $\arctan x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- For all x in $\mathbb{R}$ we have that

$$\tan(\arctan x) = x$$

- For all x in $(-\frac{\pi}{2}, \frac{\pi}{2})$ we have that

$$\arctan(\tan x) = x$$

$f(x) = \arctan x$



Please find $\arcsin(\sqrt{2}/2)$.

Please find $\arcsin(\sqrt{2}/2)$.

Solution:

We want to find an angle θ with $\sin(\theta) = \sqrt{2}/2$ and $-\pi/2 \leq \theta \leq \pi/2$. Why?

Please find arcsin($\sqrt{2}/2$).

Solution:

We want to find an angle θ with sin(θ) $= \sqrt{2}/2$ and $-\pi/2 \leq \theta \leq \pi/2$. Why? Because that is the range of arcsin(x).

Please find arcsin($\sqrt{2}/2$).

Solution:

We want to find an angle θ with sin(θ) $= \sqrt{2}/2$ and $-\pi/2 \leq \theta \leq \pi/2$. Why? Because that is the range of arcsin(x). OK fine, $\theta = \pi/4$.

Please find $\arctan(\sqrt{3})$.

Please find arctan($\sqrt{3}$).

Solution:

We want to find an angle θ with $\tan(\theta) = \sqrt{3}$ and $-\pi/2 \leq \theta \leq \pi/2 \ldots$

Please find arctan($\sqrt{3}$).
Solution:
We want to find an angle θ with $\tan(\theta) = \sqrt{3}$ and $-\pi/2 \leq \theta \leq \pi/2 \ldots \theta = \pi/3$.

Please find $\arcsin(\sin(3\pi/4))$.

Please find arcsin(sin($3\pi/4$)).

Solution:

These functions are inverses of each other, so the answer is obviously $3\pi/4$ right?

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Solution:

These functions are inverses of each other, so the answer is obviously $3\pi/4$ right? No! No! No! No! No! No! No! Why? Because $3\pi/4$ is not in the range of arcsin(x)!

Please find $\arcsin(\sin(3\pi/4))$.

Solution:

These functions are inverses of each other, so the answer is obviously $3\pi/4$ right? No! No! No! No! No! No! No! Why? Because $3\pi/4$ is not in the range of $\arcsin(x)$! With this in mind we see that

$$\arcsin(\sin(3\pi/4)) = \arcsin(\sqrt{2}/2) = \pi/4.$$

Please find arcsin(sin($3\pi/4$)).

Solution:

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$$\arcsin(\sin(3\pi/4)) = \arcsin(\sqrt{2}/2) = \pi/4.$$

UGH!

Practice Using the Trigonometry Review Sheet!

Practice Using the Trigonometry Review Sheet!
`https://math.dartmouth.edu/~m1f16/MATH1Docs/TrigonometryReview.pdf`

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It is nicely organized IMHO.

Please find $\sin(\arctan(\sqrt{3}))$.

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Solution:

$$\sin(\arctan(\sqrt{3})) = \sin(\pi/3) = \sqrt{3}/2.$$

Please find $\tan(\arcsin(12/13))$.

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Solution: This is a bit more difficult since we can't find θ directly.

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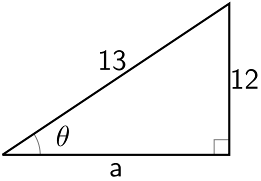

What is *a*?

Please find tan(arcsin(12/13)).

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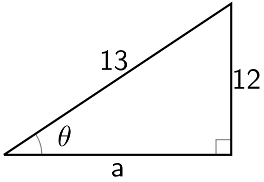
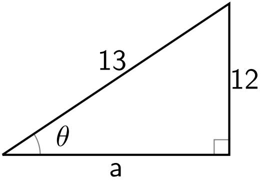


What is a? Using Pythagoras we see that $a = 5$.

Please find tan(arcsin(12/13)).

Solution: This is a bit more difficult since we can't find θ directly. However, if we let $\theta = \arcsin(12/13)$ we can draw a helpful picture.



What is a? Using Pythagoras we see that $a = 5$. Now we just have to find $\tan\theta$ for this particular (unknown) θ.

Please find tan(arcsin(12/13)).
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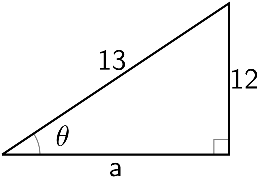

What is a? Using Pythagoras we see that $a = 5$. Now we just have to find $\tan\theta$ for this particular (unknown) θ. We see that $\tan\theta = 12/5$.

Please find a formula for $\cos(\arctan(x))$.

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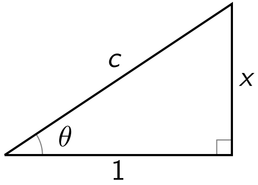
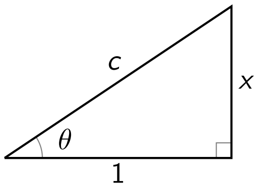


What is c?

Please find a formula for $\cos(\arctan(x))$.

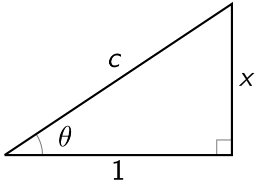
Solution: Again, we can't find θ directly. However, if we let $\theta = \arctan(x)$ we can draw a helpful picture.



What is c? Using Pythagoras we see that $c = \sqrt{x^2 + 1}$.

Please find a formula for $\cos(\arctan(x))$.

Solution: Again, we can't find θ directly. However, if we let $\theta = \arctan(x)$ we can draw a helpful picture.



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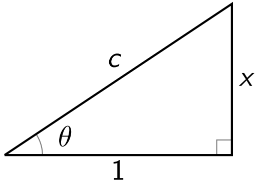

What is c? Using Pythagoras we see that $c = \sqrt{x^2+1}$. Now we just have to find $\cos\theta$ for this particular (unknown) θ. We see that $\cos\theta = 1/\sqrt{x^2+1}$.

Have a great weekend! If you get bored, think about finding all solutions to $\sin x = 0$ or $\tan x = 1 \ldots$