Math 1 Exam 1 Exercises Thursday 09-29-16

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$$25^{(5^x)} = 125^{(25^x)}.$$

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Solution:

$$\begin{aligned}
25^{(5^x)} = 125^{(25^x)} &\implies \log_5\left(25^{(5^x)}\right) = \log_5\left(125^{(25^x)}\right) \\
&\implies 5^x \log_5(25) = 25^x \log_5(125) \\
&\implies 5^x \cdot 2 = 25^x \cdot 3 \\
&\implies \log_5(5^x \cdot 2) = \log_5(25^x \cdot 3) \\
&\implies \log_5(2) + \log_5(5^x) = \log_5(3) + \log_5(25^x) \\
&\implies \log_5(2) + x = \log_5(3) + x \log_5(25) \\
&\implies \log_5(2) + x = \log_5(3) + 2x \\
&\implies x = \log_5(2) - \log_5(3).
\end{aligned}$$

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$$\begin{aligned}
\log_2(x-2)+\log_2(x+5)=3 &\implies \log_2((x-2)(x+5))=3\\
&\implies \log_2(x^2+3x-10)=3\\
&\implies 2^{\log_2(x^2+3x-10)}=2^3\\
&\implies x^2+3x-10=8\\
&\implies x^2+3x-18=0\\
&\implies (x+6)(x-3)=0.
\end{aligned}$$

So this yields two possibilities for x. But notice that only one of these choices lands in the domain of $\log_2$!

Solve the equation

$$\log_{x^2}(4) = 1.$$

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Solution:

$$\begin{aligned}\log_{x^2}(4) = 1 &\implies (x^2)^{\log_{x^2}(4)} = (x^2)^1 \\ &\implies 4 = x^2 \\ &\implies x = \pm 2.\end{aligned}$$

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What about $f(x) =$ quadratic?
What about $f(x) = 1/x$?
What about $f(x) = \sqrt[3]{x}$?

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Solution:

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$$\begin{aligned} -5 \leq f^{-1}(x+2) \leq 10 &\implies -15 \leq 3f^{-1}(x+2) \leq 30 \\ &\implies -15 - 11 \leq 3f^{-1}(x+2) - 11 \leq 30 - 11 \\ &\implies -26 \leq 3f^{-1}(x+2) - 11 \leq 19. \end{aligned}$$

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