MATH 1 LECTURE 8 WEDNESDAY 09-28-16

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Contents

I. Reminders/Announcements

start 10:10am Bartlett 105

Remarks

- Written HW#2 due
- WebWork HW06extra due
- MIDTERM1 is Thursday and covers material through exp/log. . . NO TRIG. We have shifted things slightly. . .

II. Exam Preparation

10:15am

II.1. Sequences.

- bounded, increasing, decreasing

II.2. Even/Odd Functions.

II.3. Average Rate of Change on an Interval.

II.4. **Domain and Range of a Function.**

- Compositions

Examples

Let $f(x) = \sqrt{x-6}$, $g(x) = 1$, $h(x) = 5x^2$. Find the domain and range of $h \circ f$.

Solution. $(h \circ f)(x) = 5(\sqrt{x-6})^2$. Since $5(\sqrt{x-6})^2$ simplifies to $5(x-6)$ we are tempted to say the domain and range are $\mathbb{R}$. However, the domain of $h \circ f$ cannot be larger than the domain of f which is $[6, \infty)$. Thus, the domain of $h \circ f$ is $[6, \infty)$. The range of $h \circ f$ is $[(h \circ f)(6), \infty) = [0, \infty)$ since $h \circ f$ is an increasing function.

- Inverses

Examples

Suppose f has domain $[-2, 5]$ and range $[-7, 2]$. Suppose also that f is injective on this domain. Then f^{-1} is well-defined. What are the domain and range of f^{-1}.

Solution. The domain of f^{-1} is the range of f, that is, $[-7, 2]$. The range of f^{-1} is the domain of f, that is, $[-2, 5]$.

Examples

Let $f(x) = (x+5)^2 - 2$.

(a) Find the largest domain where f is injective.

Solution. One choice is $[-5, \infty)$. Once we restrict the domain of f we are viewing f as a function $f : [-5\infty) \to [-2, \infty)$.

(b) Find the inverse of f on $[-5, \infty)$. Compute the domain and range of f^{-1}.

Solution. $f^{-1}(x) = \sqrt{x+2} - 5$. We can now verify that the domain of f^{-1} is $[-2, \infty)$ and the range of f^{-1} is $[-5, \infty)$ as we would suspect. $f^{-1} : [-2, \infty) \to [-5, \infty)$.

- Behavior under tranformations

Examples

Let's take the example from Quiz2. Let f have domain $[-1, 2]$ and range $[-2, 3]$. Consider $g(x) = (-1) \cdot f(x-3)$. Find the domain and range of g.

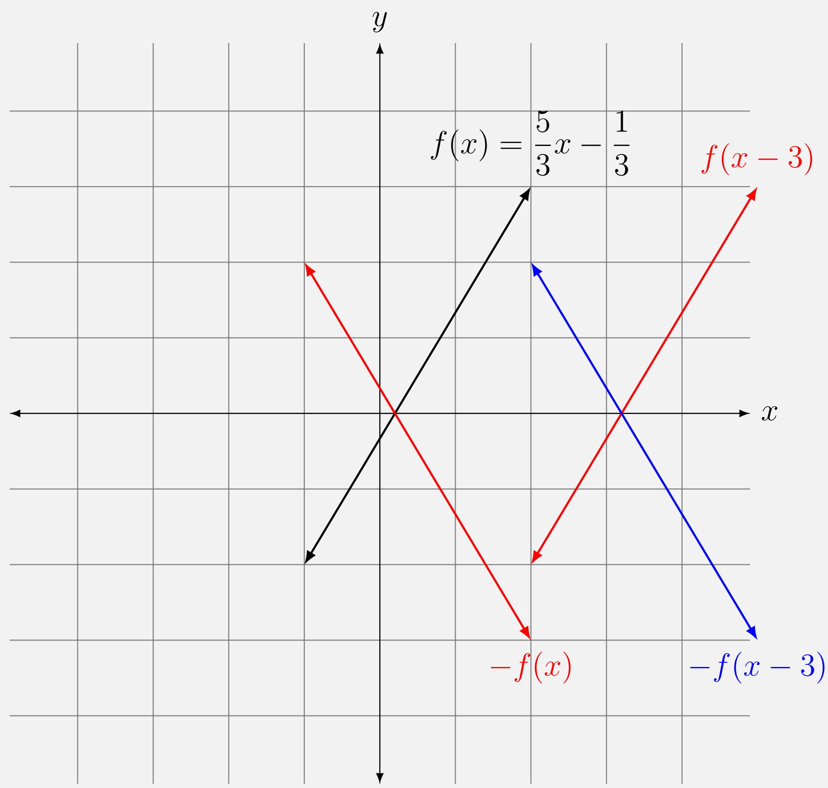
Solution. First note that to see how the domain behaves we just need $x - 3$ to land in the domain of f. Thus we are required to solve the inequality

$$-1 \leq x - 3 \leq 2 \implies 2 \leq x \leq 5.$$

That is, the domain of g is $[2, 5]$. Now note that on the domain $[2, 5]$, the function $f(x-3)$ has the same range as f on the domain $[-1, 2]$, so we just need to consider what happens with the reflection. Again, we solve an inequality

$$-2 \leq f(x-3) \leq 3 \implies 2 \geq (-1) \cdot f(x-3) \geq -3.$$

That is, the range of g is $[-3, 2]$. Alternatively, we could just figure this out geometrically as well. An example of a function f satisfying the domain and range conditions is the line through the points $(-1, -2)$ and $(2, 3)$. That is, $f(x) = (5/3)x - (1/3)$. Here's a picture to make it clear what is going on.



Examples

Let's do a more complicated example. Let f have domain $[-4, 5]$ and range $[-2, 4]$. Find the domain and range of $g(x) = (-1/2)f(3x-1)$.

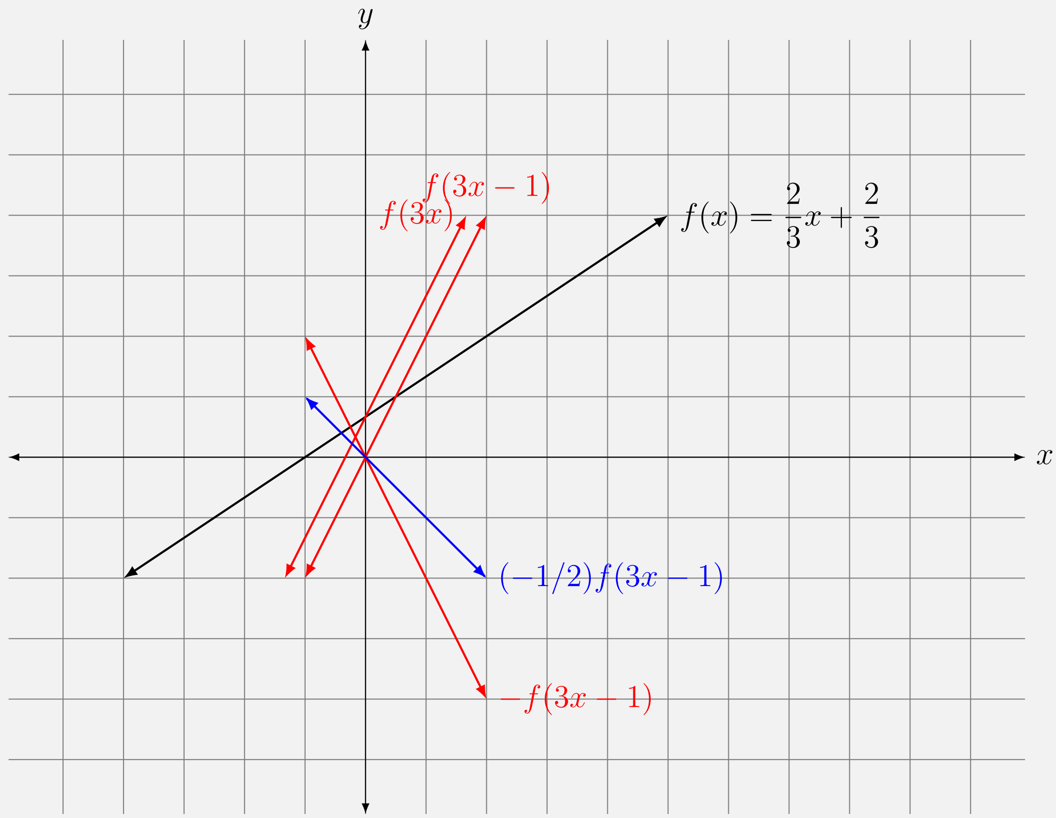
Solution. As in the previous problem, we solve the inequality

$$\begin{aligned} -4 \leq 3x - 1 \leq 5 &\implies -3 \leq 3x \leq 6 \\ &\implies -1 \leq x \leq 2. \end{aligned}$$

That is, the domain of g is $[-1,2]$. For the range we note that on $[-1,2]$, $f(3x-1)$ has the same range as f on $[-4,5]$. Thus

$$-2 \le f(3x-1) \le 4 \implies \left(-\frac{1}{2}\right)(-2) \ge \left(-\frac{1}{2}\right) f(3x-1) \ge \left(-\frac{1}{2}\right) 4$$
$$\implies 1 \ge (-1/2)f(3x-1) \ge -2.$$

That is, the range of g is $[-2,1]$. Alternatively, we could just use the picture again. The line through $(-4,-2)$ and $(5,4)$ is given by the function $f(x) = (2/3)x + (2/3)$. Here's the picture.



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II.5. **Inverse Functions.**

- one-to-one on an interval

II.6. **Lagrange Interpolation.**

II.7. **Classes of Functions.**

- linear, power, poly, rational

II.8. **exp/log.**

- definitions
- properties

- solve equations

Examples

Solve $25^{(5^x)} = 125^{(25^x)}$ for x.
Solve $\log_2(x-2) + \log_2(x+5) = 3$ for x.
Solve $\log_{x^2}(4) = 1$ for x.

11:00am

III. Trig

With any time that remains start Trig!

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end 11:15am