MATH 1 LECTURE 6 FRIDAY 09-23-16

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Contents

I. Reminders/Announcements

start 10:10am Bartlett 105

Remarks

- Written HW#1 in Kemeny 1st floor
- Written HW#2 due Wednesday
- WebWork HW05 due today
- WebWork HW06 due Monday
- Quiz Monday
- See m1f15 for old exams
- MIDTERM1 is NEXT Thursday and covers material through Monday (Trig)
- HAND OUT WORKSHEET FOR THE DAY

II. One-to-One Functions

10:15am

Definition

A function $f : D \to \mathbb{R}$ is one-to-one (injective) if

$$a \neq b \implies f(a) \neq f(b)$$

for all $a, b \in D$.
MM: [in words "f attains values in its range at most once"]
MM: [also "horizontal line test"...that's a thing]

Examples

MM: [odd roots]
MM: [find domain(s) where quadratic is injective]
MM: [some nice pictures]
MM: [maybe prove something is injective...]

Remarks

- injectivity depends on the domain

10:25am

III. Inverse Functions

Definition

Let $f : D \to \mathbb{R}$ be a function with range R. The inverse function of f is the fuction $f^{-1} : R \to D$ defined by

$$f^{-1}(y) = x \iff f(x) = y$$

for any $y \in \mathbb{R}$. It satisfies the properties:

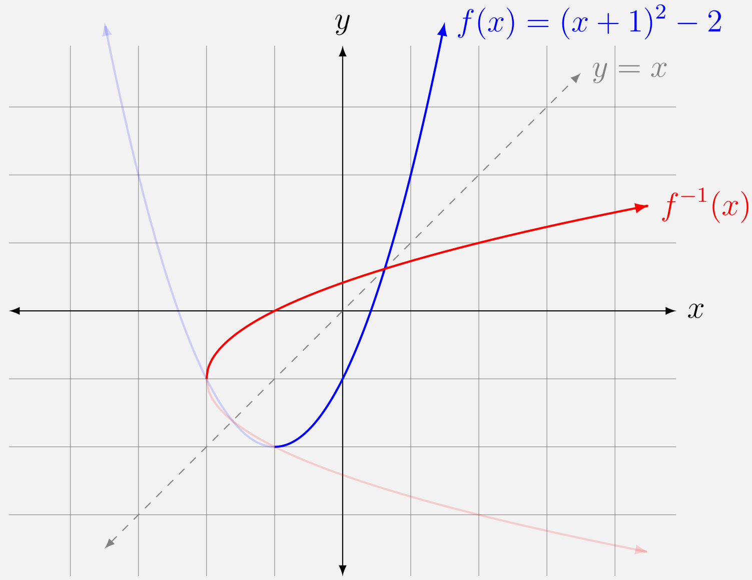
$$(f \circ f^{-1})(y) = y$$
$$(f^{-1} \circ f)(x) = x$$

for all $x \in D$ and all $y \in R$.

Examples

$$F = \frac{9}{5} \cdot C + 32$$
$$C = \frac{5}{9} \cdot (F - 32)$$

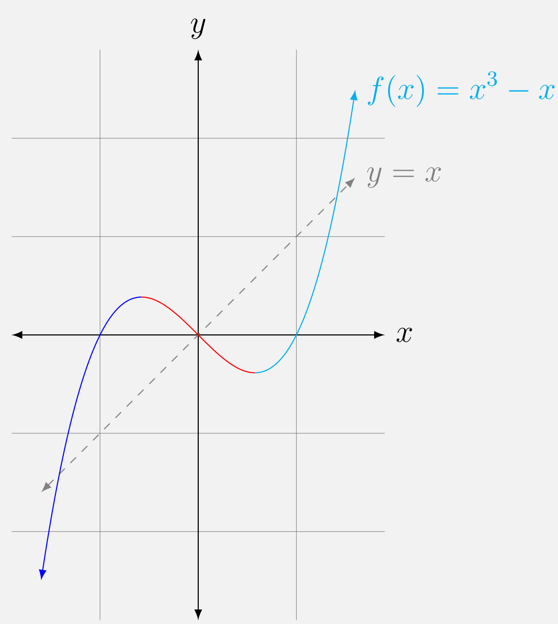
Examples



Remarks

- Geometrically the graph of $f^{-1}(x)$ is the reflection of the graph of $f(x)$ along the line $y = x$.
- To find the inverse $f^{-1}(x)$ algebraically we solve the equation $y = f(x)$ for x and then switch $x \leftrightarrow y$.

Remarks



IV. Exponential/Logarithmic Functions

10:40am

Definition

Let $a > 0$ be fixed. We define the exponential function $f(x) = a^x$.
MM: [What is the domain and range of this function?]
MM: [Why do we insist that $a > 0$?]
Now define the logarithmic function $f(x) = \log_a(x)$ by the rule:

$$y = a^x \iff \log_a(y) = x.$$

MM: [What is the domain and range of this function?]
MM: [How is $\log_a(x)$ related to a^x]

Examples

MM: [draw some example graphs]
MM: [There is really just one base $a = e = 2.7182818284590\ldots$]

Examples

Let $x, y \in \mathbb{R}$ and $a > 0$. Then

- $a^{x+y} = a^x a^y$
- $a^{x-y} = \dfrac{a^x}{a^y}$
- $(a^x)^y = a^{xy}$
- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a\left(\dfrac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^y) = y \log_a(x)$

Remarks

We are justified in picking a distinguished logarithmic function because every other one can be written as a constant multiple...

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

V. Solving Exponential/Logarithmic Equations

11:00am

Examples

Solve $e^{5x+4} = 7$ for x.

Solution. $x = \dfrac{\log_e(7) - 4}{5}$.

Examples

Solve $(e^{3x})^2 = 5e^{2x}$ for x.

Solution. $x = \dfrac{\log_e(5)}{4}$.

Examples

Solve $\log_3(x^2) = 4$ for x.

Solution. $x = \pm 9$.

MM: [if any time remains finish worksheet and/or start trig]

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end
11:15am