MATH 1 LECTURE 4 MONDAY 09-19-16

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Contents

I. Reminders/Announcements

start 10:10am Bartlett 105

Remarks

- Written HW#1 due Wednesday
- WebWork HW04 due Monday

II. Quiz

10:15am

Remarks

Quiz needs to be handed in at 10:25am.

10:25am

III. Modeling with Functions

Definition

A mathematical model is an attempt at taking a physical phenomenon and representing it mathematically.

Examples

heartrate example

Definition

A variable y is directly related to x if there is a constant $k \neq 0$ such that $y = kx$.

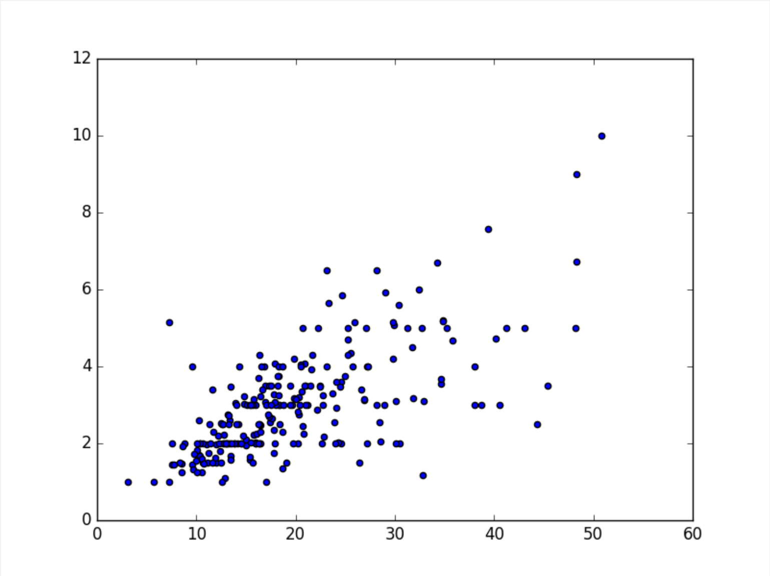
A variable y is inversely related to x if there is a constant $k \neq 0$ such that $y = k\frac{1}{x}$.

The constant k is called the constant of proportionality.

Here x is the independent variable and y is the dependent variable.

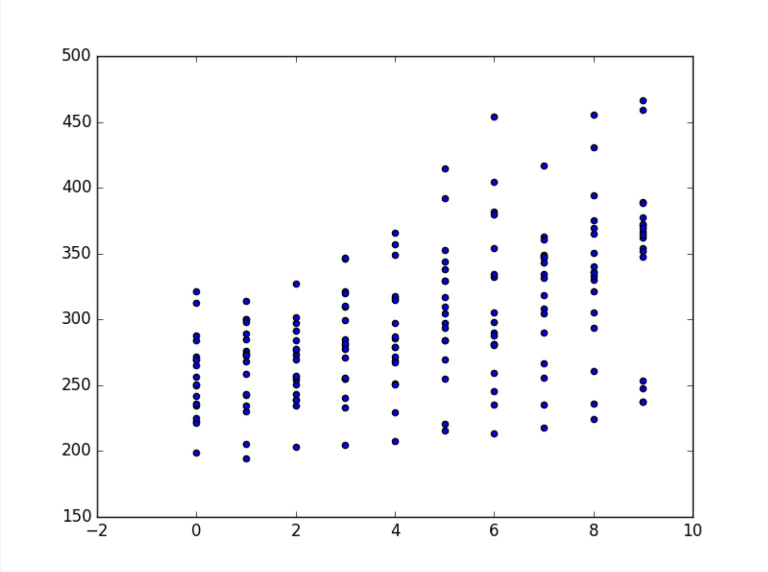
Examples

Tips



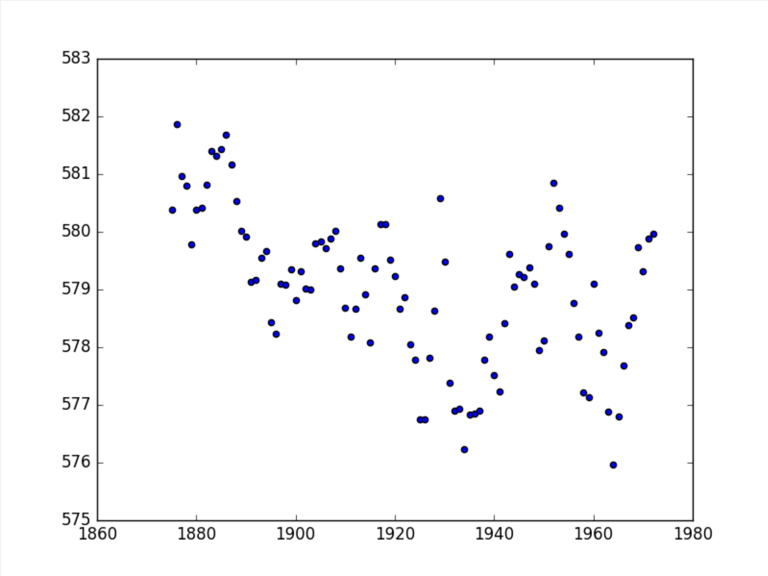
Examples

Reaction Time v. Days of Sleep Deprivation



Examples

Water level of Lake Huron



Examples

Directly Related

- Circumference v. Radius
- Brainstorm

Examples

Inversely Related

- brainstorm some examples

Examples

1 km = 0.621371 miles
1 mile = 1.60934 km

Examples

$C = \frac{5}{9}(F - 32)$

Exercises

- Convert 5 km to miles
- Convert 26.2 miles to km

Examples

The relationship can be more complicated! MM: [linear, quadratic, polynomial, exponential, trig, directly/inversely related]

10:50am

IV. Lagrange Interpolation

Definition

Extrapolation is when a mathematical model allows us to predict new information.
Interpolation is when we use a mathematical model to fill in missing information.

Examples

Lagrange Interpolation
Given $n + 1$ data points $(x_1, y_1), (x_2, y_2), \ldots, (x_{n+1}, y_{n+1})$ we would like to construct a polynomial of degree n that "hits" all the data points.

- Suppose we have 2 data points (x_1, y_1) and (x_2, y_2). Let
$$f(x) = y_1 \frac{x - x_2}{x_1 - x_2} + y_2 \frac{x - x_1}{x_2 - x_1}.$$
Then $f(x_1) = y_1$ and $f(x_2) = y_2$ as desired.
- Suppose we have 3 data points $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3). Let
$$f(x) = y_1 \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} + y_2 \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} + y_3 \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}.$$
Then $f(x_i) = y_i$ for $i \in \{1, 2, 3\}$.

Exercises

- Suppose at the beginning of the month (day 1) you have 145 USD in your bank account and at the end of the month (day 30) you have 58 USD. Use Lagrange interpolation to estimate how much you have in the bank on day 23.

  Solution. The data points are $(1, 145)$ and $(30, 58)$. Lagrange interpolation yields the line
$$\begin{aligned} f(x) &= 145 \frac{x - 30}{1 - 30} + 58 \frac{x - 1}{30 - 1} \\ &= 145 \frac{x - 30}{-29} + 58 \frac{x - 1}{29} \\ &= -5(x - 30) + 2(x - 1) \\ &= -3x + 150 - 2 \\ &= -3x + 148. \end{aligned}$$
  Now $f(23) = 79$.

- Now suppose we also know that on day 2 we still have 145 USD. Use Lagrange interpolation to estimate how much we have in the bank on day 23.

  Solution. The data points are $(1, 145), (2, 145), (30, 58)$. Lagrange interpolation yields the polynomial
$$\begin{aligned} f(x) &= 145 \frac{(x - 2)(x - 30)}{(1 - 30)(1 - 2)} + 145 \frac{(x - 1)(x - 30)}{(2 - 1)(2 - 30)} + 58 \frac{(x - 1)(x - 2)}{(30 - 1)(30 - 2)} \\ &= 5(x - 2)(x - 30) - \frac{145}{28}(x - 1)(x - 30) + \frac{1}{14}(x - 1)(x - 2). \end{aligned}$$
  Now $f(23) = 95.5$.

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end
11:15am