Final Exam Review 2

These problems are intended to reflect **some** of the skills and material you should have learned in Math 1. The material covered here is **NOT** comprehensive. Moreover, these are **NOT** intended to reflect the difficulty of problems you will see on the exam.

(1) Derivative formulas

1. What are the different methods that we have used to find derivative formulas? Give an example for each.
2. Fill in the entries with the respective derivatives below:
   - $\frac{d}{dx}\sin(x) =$
   - $\frac{d}{dx}\cos(x) =$
   - $\frac{d}{dx}\tan(x) =$
   - $\frac{d}{dx}\sec(x) =$
   - $\frac{d}{dx}\csc(x) =$
   - $\frac{d}{dx}\cot(x) =$

   - $\frac{d}{dx}e^x =$
   - $\frac{d}{dx}a^x =$
   - $\frac{d}{dx}\ln(x) =$
   - $\frac{d}{dx}\log_a(x) =$

   - $\frac{d}{dx}\arcsin(x) =$
   - $\frac{d}{dx}\arccos(x) =$
   - $\frac{d}{dx}\arctan(x) =$
3. Are there any derivatives that we've discussed that are missing from this list?
4. Choose one of the following and prove its derivative formula using only formulas *higher* in the list: $\csc(x)$, $\log_a(x)$, or $\arctan(x)$.

(2) Derivative rules

1. What are the different derivative rules? You should have a list of 8 rules. Write down each one using the functions f and g.
2. Find a function whose derivative can be found with either the power rule *or* the quotient rule.
3. Find a function whose derivative "requires" use of the quotient rule.
4. Find a function whose derivative can be found with either the product rule *or* the chain rule.
5. Find a function whose derivative requires use of the chain rule.
6. Find the derivative of $\dfrac{\cos(3x)e^x}{\sin(3x)}$.
7. Find the derivative of $\dfrac{x^3+2x^2+x}{x+1}$.

(3) Tangent lines

1. What is a tangent line? How do you find the slope of the tangent line?
2. Do all functions have tangent lines at all points? Explain or give an example.
3. What are two different ways that we have used tangent lines in class?

(4) Implicit differentiation

1. What is an implicit equation? How do they differ from functions?
2. Find y' for the equation $y^2\cos(x) = e^y$.
3. Find y' for the equation $e^y = x^3 + 2x - 1$ in two ways:
   (a) Solve for y *first* and then find y'.
   (b) Use implicit differentiation.
4. Find the tangent line for $x^2\cos(y) = y^2\sin(x)$ at the point $\left(\pi, \dfrac{\pi}{2}\right)$.

(5) Newton's method

1. What is the purpose of Newton's Method?
2. Explain Newton's Method (i.e., how to use it) and be sure to write down the formula.
3. Use Newton's Method to approximate $\sqrt{7}$ out to 4 digits. Do as much by hand as reasonable. (Hint: Start at $x_0 = 3$. Why is this a good starting point?)

(6) Taylor polynomials

1. Define the nth-order Taylor polynomial for f at a.
2. How is a Taylor polynomial used?
3. What is the 5th-order Taylor polynomial for $x^3 + 2x - 2$ at $a = 1$? Simplify your answer. Are you surprised by it? Explain.
4. What is the 4th-order Taylor polynomial for $\cos(x) + e^x$ at $a = 0$?
5. What is the 4th-order Taylor polynomial for $\sin(3x)$ at $a = 0$.

(7) Sequences

1. For each of the following properties, write a formula for a sequence with that property:

   a) Alternating
   b) Increasing
   c) Decreasing
   d) Bounded
   e) Unbounded
   f) Convergent
   g) Divergent

2. Write the first 5 terms of the following sequences:

   a) $a_n = 2a_{n-1} + 3a_{n-2} - 4 \qquad a_1 = 2 \qquad a_2 = 4$
   b) $\left\{ \dfrac{n^2+1}{2n-1} \right\}_{n=1}^{\infty}$
   c) $a_n = \dfrac{(-1)^n 2^n}{n+1}$

3. Prove that $\displaystyle\lim_{n\to\infty} \left(1 - \frac{1}{2^n}\right) = 1.$
4. Prove that the limit of $\left\{ \dfrac{3n^2-1}{2n^2+1} \right\}_{n=1}^{\infty}$ is $\dfrac{3}{2}$.

(8) Linearization and Differentials

1. What is the purpose of computing the linearization of a function?
2. Define the linearization of a function f at a point a.
3. Define the differential of a function f at a point a.
4. What is the difference between Δy and dy?
5. Compute the linearization of $f(x) = \sqrt[3]{x+2}$ at $a = 6$ and use it to estimate $f(6.1)$ and $f(5.9)$.
6. Compute the linearization of $f(x) = x^3 - 2x^2 + 5x - 1$ at $a = 1$ and use it to estimate $f(.8)$ and $f(1.2)$.
7. Compute Δy and dy of $f(x) = e^x + x^2$ at $a = 0$ with $dx = .1$ and $dx = -.5$.

(9) L'Hospital's Rule

1. Why do we need to check that a limit satisfies an indeterminant form before applying L'Hospital's rule?
2. Write 8 possible indeterminant forms.
3. Rewrite the following limits as one of the indeterminant forms you listed above:

   a) $\lim_{x\to\infty} x^3 e^{-x^2}$

   b) $\lim_{x\to 0^+} \sin(x)\ln(x)$

   c) $\lim_{x\to 0} \cot(2x)\sin(6x)$

4. Compute the following limits:

   a) $\lim_{x\to\infty} \dfrac{e^{0.1x}}{x^3}$

   b) $\lim_{x\to 0} \dfrac{x^2}{1-\cos(x)}$

   c) $\lim_{x\to\frac{\pi}{2}} \dfrac{\sec(x)}{\tan(x)}$

   d) $\lim_{x\to\infty} \dfrac{\ln(x)}{\sqrt{x}}$