FINAL EXAM REVIEW 1

These problems are intended to reflect **some** of the skills and material you should have learned in Math 1. The material covered here is **NOT** comprehensive. Moreover, these are **NOT** intended to reflect the difficulty of problems you will see on the exam.

(1) Solving expressions

1. Write down the 3 logarithm laws and the 3 exponential laws.
2. Simplify $\left(\dfrac{x^2y^{1/2}}{z^{-1}}\right)^{2/3}$
3. Solve $e^{2y-1} = \cos(x)$ for y.
4. Solve $2\log_3(y) = e^x$ for y.
5. Find a formula for $\arctan(\cos(x))$.

(2) Rates of change

1. What is the average rate of change of f over $[a, b]$? What is it geometrically? What does it mean?
2. What is the instantaneous rate of change of f at a? What is it geometrically? What does it mean?
3. How are average and instantaneous rates of change related?

(3) Find the following limits. A (∗) will indicate that you should be able to do this in two different ways.

1. $\lim_{x\to 2}(x^{-2} + 2)$
2. $\lim_{x\to -1}(-3x - e^{-x})$
3. (∗) $\lim_{x\to 0}\dfrac{\sqrt{1-x}-1}{x}$
4. (∗) $\lim_{x\to 3}\dfrac{x^2-7x+12}{x-3}$
5. $\lim_{x\to -1}\dfrac{\ln(x+2)}{x+1}$
6. $\lim_{x\to \pi}\cot(x)$

(4) Continuity

1. What is the definition of f being continuous at a? How do you verify that a function is continuous at a?
2. Describe continuous functions intuitively (i.e., without using specific math terminology).
3. What functions are continuous on their domains?
4. Is the function $g(x) = \begin{cases} x^2 - 2 & x < 1 \\ 3 + 2e^x & x \geq 1 \end{cases}$ continuous at $x = 1$?
5. Is the function $h(x) = \begin{cases} \cos(x) & x < 0 \\ x + 1 & x \geq 0 \end{cases}$ continuous at $x = -1$?

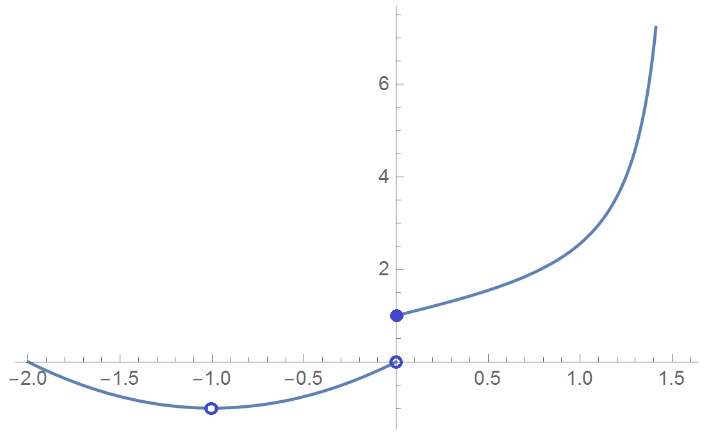
(5) Discontinuities

1. What are the different types of discontinuities? Draw a picture of each and then write down the limit/formal definition of each type of discontinuity.
2. Write down different functions, one with each type of discontinuity.

(6) Derivatives

1. Write down the limit definition of the derivative.
2. If you know that f is continuous at a, must f be differentiable at a?
3. Use the limit definition to find the derivatives of $x^2 + 3$ and $\sqrt{x} - 1$.
4. Find the tangent line to $f(x) = 6x^2 - 2$ at $x = 4$.

(7) Let $f(x) = \begin{cases} \dfrac{x^3+3x^2+2x}{x+1} & x < 0 \\ \tan(x) & 0 \leq x < \dfrac{\pi}{2} \end{cases}$. The graph of f is



1. What is the domain of f?
2. Find the intervals on which f increasing and decreasing.
3. Is f one-to-one?
4. Find
   (a) $\lim_{x\to 0^+} f(x)$
   (b) $\lim_{x\to -1} f(x)$
   (c) $\lim_{x\to \frac{\pi}{2}^-} f(x)$
   (d) $\lim_{x\to \frac{\pi}{4}} f(x)$
   (e) $\lim_{x\to 1} f(x)$
5. Find the vertical asymptotes of f (if it has any).
6. Where is f continuous? Alternatively, where is f discontinuous?
7. For each discontinuity of f, determine the type of discontinuity.
8. Where is f differentiable?
9. What is $f'(x)$?

Note: For parts 4-8, you should be able to do this **both** with the graph and with only the function expression (i.e., no graph).