Math 1: Calculus with Algebra
Fall 2015

Quiz 3

Name: Answer Key

Instructions: (24 points) This quiz consists of 5 problems covering material from the 3rd and 4th weeks of class. Credit is awarded for correct solutions in which you **show your work**. You will have 30 minutes to complete this quiz. You may not use a calculator, textbook, notes, or any outside source while taking this quiz.

(5^{pts}) **1.** Find the inverses of each of the following functions:

(a) $f(x) = 3^{2x-1}$

Solution: Let $y = f(x) = 3^{2x-1}$. Applying $\log_3$ to both sides, we get

$$\log_3(y) = \log_3(3^{2x-1})$$

Since 3^x and $\log_3(x)$ are inverses, this simplifies to

$$\log_3(y) = 2x - 1.$$

Solving for x we get $f^{-1}(y) = x = \dfrac{\log_3(y) + 1}{2}$.

(b) $g(x) = 2\ln(x + 1)$

Solution: Let $y = g(x) = 2\ln(x + 1)$. Dividing by 2, we get

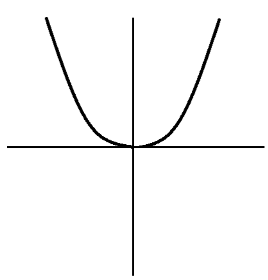
$$\frac{y}{2} = \ln(x + 1).$$

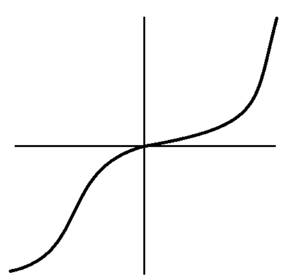
Now we can exponentiate both sides to get

$$e^{\frac{y}{2}} = e^{\ln(x+1)} = x + 1.$$

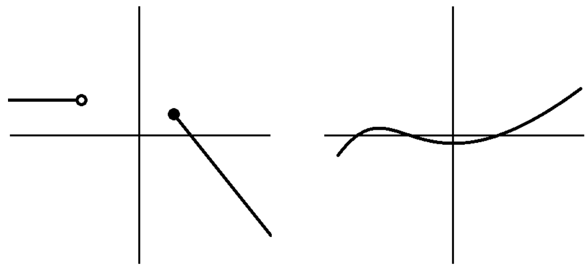
So $g^{-1}(y) = e^{\frac{y}{2}} - 1$.

(5^{pts}) **2.** For each of the following graphs, determine whether the function is one-to-one (YES or NO). If it is one-to-one, draw the inverse function on the same graph.





A. NO.   B. YES.



C. NO. D. NO.

(4pts) **3.** (Angles)

(a) Convert $\frac{5\pi}{6}$ radians to degrees.

Solution:

$$\frac{5\pi}{6} \text{ rad} \cdot \frac{180 \text{ deg}}{\pi \text{ rad}} = 5 \cdot 30 \text{ deg} = 150 \text{ deg}$$

(b) Convert -300 degrees to radians.

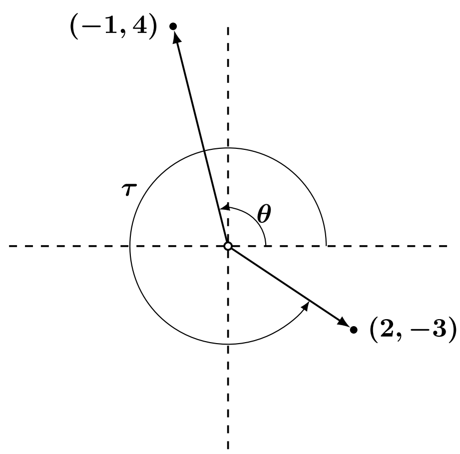
Solution:

$$-300 \text{ deg} \cdot \frac{\pi \text{ rad}}{180 \text{ deg}} = \frac{-30\pi}{18} \text{ rad} = -\frac{5\pi}{3}.$$

(c) Draw each of these angles in standard position.

Solution: .

(5pts) **4.** Compute the values of the functions using the information below. (Remember, you can use a triangle if you interpret the length of the sides correctly.)



(a) $\sin(\theta)$

Solution: To figure out $\sin(\theta)$, we need to know the length of the line segment from $(0,0)$ to $(-1,4)$. This is found by using the Pythagorean theorem: $\ell^2 = (-1)^2 + 4^2 = 17$. So $\ell = \sqrt{17}$.

Now $\sin(\theta)$ is the y-coordinate of the point $(-1,4)$ divided by the length ℓ. So $\sin(\theta) = \frac{4}{\sqrt{17}}$.

Alternatively, we can draw out the triangle where the opposite side has length 4 (as the sine function should be positive in the second quadrant).

(b) $\sec(\tau)$

Solution: In this case $\ell = \sqrt{2^2 + (-3)^2} = \sqrt{13}$. Since $\sec(\tau) = \frac{1}{\cos(\tau)}$, we can find $\cos(\tau)$ to finish out the computation.

Here $\cos(\tau) = \frac{2}{\sqrt{13}}$ (the x-coordinate divided by the length of the line) and so $\sec(\tau) = \frac{\sqrt{13}}{2}$.

(5[pts]) **5.** Prove that the limit of the sequence $\left\{\frac{2n^2-1}{n^2-1}\right\}_{n=2}^{\infty}$ is 2.

Solution: Let $\varepsilon > 0$ be arbitrary.

Now we want to solve $\left|\frac{2N^2-1}{N^2-1} - 2\right| < \varepsilon$ for N. Getting a common denominator and simplifying, we get

$$\left|\frac{2N^2-1}{N^2-1} - 2\frac{N^2-1}{N^2-1}\right| < \varepsilon$$
$$\left|\frac{2N^2-1-2(N^2-1)}{N^2-1}\right| < \varepsilon$$
$$\left|\frac{2N^2-1-2N^2+2}{N^2-1}\right| < \varepsilon$$
$$\left|\frac{1}{N^2-1}\right| < \varepsilon$$

Since $N \geq 2$ we can get rid of the absolute values. This gives us

$$\frac{1}{N^2-1} < \varepsilon.$$

Now we can cross-multiply (both $N^2 - 1$ and ε are positive numbers, so the inequality doesn't flip) to get

$$\frac{1}{\varepsilon} < N^2 - 1.$$

Finally adding 1 to both sides and taking the square root, we have

$$\sqrt{\frac{1}{\varepsilon} + 1} < N.$$

Thus we have solved for N and can conclude that $\lim_{n\to\infty} \frac{2n^2-1}{n^2-1} = 2$ as desired.