

Final Exam
Math 1
November 22nd, 2013

Name: _____

Please circle your instructor's name below

Infeld

Petit

Please read the following instructions before starting the exam:

- This exam is closed book, with no calculators, notes, or book allowed. You may not give or receive any help during the exam, though you may ask the instructors for clarification if necessary.
- Be sure to show all work whenever possible. Even if your final answer is incorrect, we can assign an appropriate amount of partial credit if we can see how you arrived at your answer.
- Please circle or otherwise indicate your final answer, if possible.
- The test has a total of 10 questions, worth a total of 150 points. Point values are indicated for each question.
- You will have three hours from the start of the exam to complete it.
- Good luck!

HONOR STATEMENT: I have neither given nor received help on this exam, and I attest that all the answers are my own work.

SIGNATURE: _____

This page is for grading purposes only.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

1. Circle the correct answer to each question. In each case there is only one that is correct!
No partial credit. [5 points each]

If the function f is continuous on $[0, 1]$ and differentiable on $(0, 1)$, and $f(0) = 1$ and $f(1) = 1$, then by Rolle's Theorem:

- The function f is constant on $(0, 1)$.
- The function f has a local minimum on $(0, 1)$.
- The function f has a local maximum on $(0, 1)$.
- The function f has a critical point on $(0, 1)$.

If $f(x)$ fails the horizontal line test, then:

- It's not an even function.
- It doesn't have an inverse.
- It's not a function.
- It's an odd function.

If you have the graph of $f(x)$, how would you get the graph of $2f(x + 4)$?

- Stretch vertically by 2 and shift right by 4.
- Stretch vertically by 2 and shift left by 4.
- Stretch horizontally by 2 and shift up by 4.
- Stretch horizontally by 2 and shift down by 4.

$\log_a(x)$ is:

- $\ln(x^a)$
- $\log_b(x^a)$
- $\frac{\ln(x)}{\ln(a)}$
- $\frac{\ln(a)}{\ln(x)}$

If the function f is continuous on $[0, 2]$ and differentiable on $(0, 2)$, and $f(0) = 1$ and $f(2) = 13$, then by Mean Value Theorem:

- There exists c between 1 and 2 with $f'(c) = 12$
- There exists c between 0 and 2 with $f'(c) = 6$
- There exists c between 1 and 2 with $f(c) = 6$
- There exists c between 0 and 2 with $f(c) = 13$

2. Find the domain and range of $f(x) = 5^{3x+7} + 4$ [5 points]

Find the inverse of $f(x)$. [5 points]

3. Compute the following limits [5 points each]

$$\bullet \lim_{x \rightarrow -2} \frac{-\frac{1}{4} - \frac{1}{2x}}{x+2}$$

$$\bullet \lim_{x \rightarrow +\infty} \frac{\pi x^5 - 3x^3 + x - 2}{2x^5 - 17}$$

$$\bullet \lim_{x \rightarrow 0} \frac{\cos(x) - 1 + x^2}{x^4}$$

4. Compute the derivatives of the following functions: [5 points each]

- $f(x) = \tan^{-1}(3x) \ln(x^2)$

- $f(x) = \frac{x^3 - 3 \cos(2x)}{2x}$

- $f(x) = (\sin(x))^{1/x}$

5. Find the absolute maximum and the absolute minimum of the function

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1$$

on the interval $[-2, 1]$. [15 points]

6. Recognize if the following are indeterminate forms and which type, by circling the correct answer, and find the limits: [5 points each]

• $\lim_{x \rightarrow 0} \frac{e^x + 2x - 1}{3x^2}$ is:

- an indeterminate form of type $\frac{0}{0}$
- an indeterminate form of type $\frac{\infty}{\infty}$
- an indeterminate product $\infty \times 0$
- not an indeterminate form

$$\lim_{x \rightarrow 0} \frac{e^x + 2x - 1}{3x^2} =$$

• $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$ is:

- an indeterminate form of type $\frac{0}{0}$
- an indeterminate form of type $\frac{\infty}{\infty}$
- an indeterminate product $\infty \times 0$
- not an indeterminate form

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} =$$

7. Find the values of a and b that make f continuous everywhere. [10 points]

$$f = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x < 1 \\ ax + b & \text{if } 1 \leq x \leq 2 \\ 3b & \text{if } 3 < x \end{cases}$$

8. This problem will have you graph the function

$$f(x) = x^3 - 12x^2 + 36x$$

The question is worth 25 points overall.

- Find the domain of $f(x)$ and the points where $f(x) = 0$. [5 points]

- Compute the limits as $x \rightarrow \pm\infty$ of $f(x)$. [5 points]

- Find the intervals where $f(x)$ is increasing, as well as any maxima/minima the function might have. [5 points]

- Find the intervals where $f(x)$ is concave up, as well as any inflection point the function might have. [5 points]

- Use the information to draw the function. [5 points]

9. Find the equation of the tangent line to the implicitly defined function

$$2 \sin(x + y) = 3x - 3y$$

at the point (π, π) . [10 points]

10. A man starts walking north at a constant rate of 4 ft/min from a point P . One minute later from the same point P a woman start walking east at a constant rate of 5 ft/min. At what rate are the two moving apart after a total of five minutes have passed since the man started walking? [15 points]