Name:_____

Section:_____

MATH 1 MIDTERM 2
November 3, 2010

INSTRUCTIONS: This is a closed book, closed notes, calculator-free, computer-free, open-mind exam. You are not to give nor to receive help from any outside source during the exam. Remember that your instructors can clarify any questions that are not clear to you.

Please show all of your work and justify all of your answers.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own work.

_____
Signature

Question	Points	Score
1	7	
2	12	
3	14	
4	8	
5	8	
6	18	
7	10	
8	6	
9	3	
10	6	
11	7	
12	1	
Total:	100	

1. (a) [3 points] For a function f, please define what it means for f to be continuous at a number a.

   (b) [4 points] Using your definition, please show that $f(x) = \dfrac{\sqrt{2x} + 6x}{2x - 1}$ is continuous at $x = 2$.

2. Please compute the following limits. Remember to state any limit laws or theorems that you use.

(a) [4 points] $\displaystyle\lim_{x\to\infty} \frac{e^{2+x}}{1+4e^x}$

(b) [4 points] $\displaystyle\lim_{x\to1} \frac{x^2+x-2}{x-1}$

(c) [4 points] $\displaystyle\lim_{x\to 2} \sin\left(\pi x - \frac{3\pi}{2}\right)$

3. (a) [3 points] Please define the derivative of a function f.

(b) [5 points] Consider $f(x) = x^2 - 3x + 9$. Now compute the derivative of f using the definition of the derivative.

(c) [2 points] Using the above, please find the derivative of f at $x = 3$.

(d) [4 points] Now compute the derivative of f using the power rule and find $f'(3)$.

4. You have differentiable functions f and g. Now please compute the derivatives of the following functions:

(a) [4 points] Consider $h(x) = 2f(x) - \dfrac{g(x)}{x}$. Then find $h'(x)$.

(b) [4 points] Consider $k(x) = (x - 3)g(x) + f(x)$. Then find $k'(x)$.

5. [8 points] Please state 4 of the limit laws precisely, including any excep-
   tions that each law has, and including at least one that has a power or
   root in it.

   a)

   b)

   c)

   d)

6. (a) [3 points] Please define a horizontal asymptote of a curve $y = f(x)$.

   (b) [3 points] Please define a vertical asymptote of a curve $y = f(x)$.

(c) Find the horizontal and vertical asymptotes of the following functions. If it does not have a horizontal or vertical asymptote, please explain why.

(a) [6 points] $f(x) = \dfrac{3x + 5}{3x^2 + 5x - 2}$

(b) [6 points] $g(x) = \dfrac{\sqrt{3x^4 + 2x^2 + 7x + 1}}{x^2 - x - 2}$

7. [10 points] Captain Continuity is going to a party at Lucy Limit's house and he would really like to meet some continuous functions there. Can you help him out by listing five kinds of functions that are continuous on their domains?

a)

b)

c)

d)

e)

8. Determine if the functions defined below are continuous. Please justify your answer.

(a) [2 points] Human population of the world as a function of time.

(b) [2 points] Volume of water in a kettle as a function of boiling time.

(c) [2 points] Cost of mailing a first-class letter as a function of weight of the letter.

9. [3 points] One of the following statements is false. Please point out the false statement and explain why it is false.

   (a) There are rational functions with no horizontal asymptote.

   (b) The function $f(x) = |x + 2| + |x - 5|$ is continuous on $(-\infty, \infty)$.

   (c) If $f(x)$ and $g(x)$ are both continuous at a, then $(f \circ g)(x) = f(g(x))$ is also continuous at a.

10. [6 points] Sketch the graph of a function f which satisfies the following conditions:

a) $\lim\limits_{x \to 1^+} f(x) = 2$

b) $\lim\limits_{x \to 1^-} f(x) = \infty$

c) $\lim\limits_{x \to -\infty} f(x) = -3$

d) $\lim\limits_{x \to \infty} f(x) = 3$

11. (a) [4 points] Please state the squeeze theorem. Make sure you include every condition we need to check for the theorem to apply.

   (b) [3 points] Now show that $\lim\limits_{x \to 0} x^4 \cos\left(\dfrac{2}{x}\right) = 0$

12. [1 point] The new math singing group "The Higher Derivatives" are looking for a title for their newest CD. They have asked you for help! Please write a name for their new album.