NAME:_____

SECTION:_____

MATH 1 MIDTERM 2 PRACTICE EXAM

October 30, 2009

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may not use a calculator.
- Show all of your work.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

_____

Signature

Question	Points	Score
1	6	
2	9	
3	6	
4	10	
5	9	
6	10	
7	8	
8	8	
9	6	
10	5	
11	8	
12	7	
13	8	
Total:	100	

1. [6 points] Suppose that f is a differentiable function and that a is a number in the domain of f. In class we learned that the *number*

$$\lim_{b \to 0} \frac{f(a+b) - f(a)}{b}$$

is the slope of the tangent line to the graph of f at the point $(a, f(a))$. Write a brief essay (3-4 sentences) explaining why this is true.

(Note: We won't ask you to write an essay on the actual exam, but you are responsible for understanding this concept!)

2. Compute the following derivatives. Pay attention to special instructions. (You may use your work in one part to complete subsequent parts.)

   (a) [4 points] Use the Quotient Rule to compute the derivative of $\cot(x)$.

   (b) [2 points] What is the derivative of $f(x) = -3x^5 - 2x^3 + \cot(x)$?

   (c) [3 points] What is the derivative of $e^x \cdot x^3$?

3. Compute the following limits. Write DNE if the limit does not exist. Each of these limits describes an asymptote of the graph. State whether each asymptote is vertical or horizontal. If it is horizontal, state which y-value it passes through. If it is vertical, state which x-value it passes through.

   (a) [2 points] $\lim_{x\to\infty} x^{-4}$

   (b) [2 points] $\lim_{x\to\infty} -3 \cdot \frac{1}{x^2}$

   (c) [2 points] $\lim_{x\to 0} \frac{1}{x}$

4. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x^3 & \text{if } x > 1 \end{cases}$$

(a) [2 points] Is $f(x)$ continuous at $x = 1$? Explain how you know. (You may sketch the graph if you find this helpful, but your answer must involve right and left limits!)

(b) [3 points] Compute $\lim_{x \to 1^-} \dfrac{f(x) - f(1)}{x - 1}$. Note: This is a **left** limit. With this in mind, be sure you are using the correct formula for $f(x)$.

(c) [3 points] Compute $\lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$. Note: This is a **right** limit. Again, be sure you are using the correct formula for $f(x)$.

(d) [2 points] Based on the last two parts, is $f(x)$ differentiable at $x = 1$? Explain how you know.

5. Compute the following derivatives. Pay attention to special instructions. (You may use your work in one part to complete subsequent parts.)

   (a) [3 points] Let $f(x) = x \cdot e^x$. Find $f'(x)$.

   (b) [3 points] Let $f(x)$ be as above and let $g(x) = x + 1$. Use the Quotient Rule to compute the derivative of $\dfrac{f(x)}{g(x)}$. You do not need to simplify further after taking the appropriate derivative(s).

   (c) [3 points] Let $f(x)$ and $g(x)$ be as above. Use the Product Rule to compute $\dfrac{d}{dx}(f(x) \cdot g(x))$.

6. There are (at least) 2009 different ways to differentiate the function $m(x) = x^{26}$.

   (a) [3 points] Calculate the derivative directly using the Power Rule.

   (b) [3 points] Observe that $f(x) = x^{26} = (x^{13})^2 = x^{13} \cdot x^{13}$. Calculate the derivative using the product rule.

   (c) [4 points] Find one more way to calculate the derivative.

7. The function $\mathrm{B}(t) = 10 \cdot e^t$ ("ten times e to the t") models the bank account of Bernie Madoff after t days.

   (a) [2 points] How much money is in Bernie's bank account at day zero (note "day zero" corresponds to $t = 0$)?

   (b) [2 points] How many days will it be until Bernie's bank account has \$40 in it?

   (c) [4 points] Assuming your answer for part b is correct, how quickly is Bernie's bank account growing when he has \$40 in it? In other words, what is the instantaneous rate of change of the amount of money in his bank account at the time value you calculated in part (b)? You may leave your answer completely unsimplified.

8. Compute the following limits. Write DNE if the limit does not exist.

(a) [2 points] $\lim_{x\to\infty} \dfrac{\sqrt{3x^2-4x+7}}{2x+5}$

(b) [2 points] $\lim_{x\to-\infty} \dfrac{2x+5}{3x^2-4x+7}$

(c) [2 points] $\lim_{x\to\infty} \dfrac{3x^2-4x+7}{(2x+5)^2}$

(d) [2 points] $\lim_{x\to\infty} \dfrac{3x^2-4x+7}{2x+5}$

9. Let $g(x) = \dfrac{x+1}{x}$.

   (a) [4 points] Compute $g'(x)$ using the limit definition of the derivative. No credit will be given for using the quotient rule.

   (b) [2 points] Use your work above to find the slope of g at $x = 1$.

10. [5 points] Find the equation of the line tangent to the curve $g(x) = x \cdot \cos(x) - e^x \cdot \sin(x)$ at the point $(0, g(0))$.

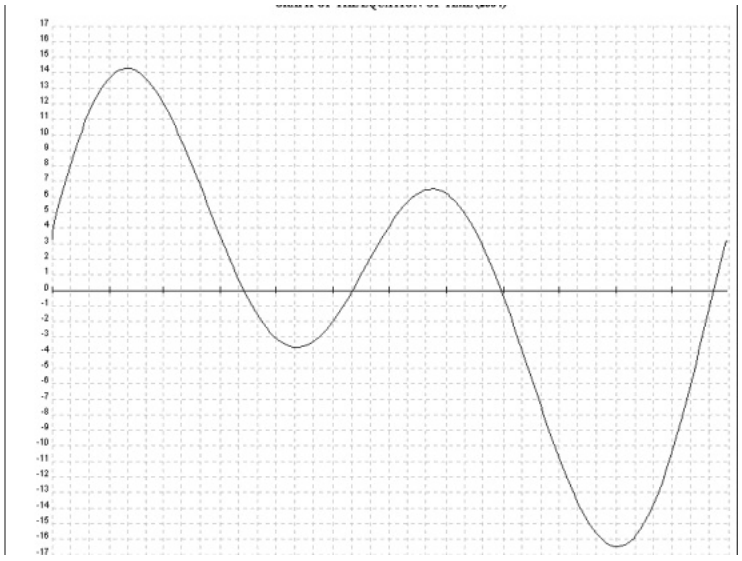
11. Suppose the velocity is given by $v(t) = -4t + 6$.

(a) [2 points] In general, what is the relationship between the position function and the velocity function?

(b) [3 points] Based on your answer to part (a), could $p(t) = -2t^2 + 5t + 12$ be the position function which has $v(t)$ as it's velocity function? Explain why or why not.

(c) [3 points] Using the function for velocity given above, find a function for acceleration.

12. [7 points] Sketch a graph of the derivative of the function pictured below:

1.jpg 

13. [8 points] In what follows, $f(x)$ and $g(x)$ are continuous and differentiable, and c is a constant. Label each statement as "true" or "false" (**you must write the whole word**). A statement is considered true only if it is always true; otherwise it is false. (one point each)

(a) ____ Assuming that $f(x)$ and $g(x)$ are both continuous,

$$\lim_{x\to a}\left(f(x)\cdot g(x)\right) = f(a)\cdot g(a).$$

(b) ____ $\left(\dfrac{f(x)}{g(x)}\right)' = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}.$

(c) ____ Assuming that $\lim_{x\to a} g(x) \neq 0$, then

$$\lim_{x\to a}\left[4f(x) - \frac{1}{g(x)}\right] = 4\cdot\lim_{x\to a} f(x) - \frac{1}{\lim_{x\to a} g(x)}$$

(d) ____ $\lim_{x\to a}\sqrt{2\cdot f(x)} = 2\cdot\sqrt{\lim_{x\to a} f(x)}.$