Name: ____________________

Section: ____________________

Math 1 Practice Final Exam

November 29, 2009

Instructions: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- You may not use a calculator.
- In order to receive full credit, you must show your work.

Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	15	
2	20	
3	6	
4	15	
5	14	
6	10	
7	16	
8	18	
9	8	
10	6	
11	14	
12	8	
Total:	150	

1. Find the following limits using any method. Write $\pm\infty$ if the limit is plus or minus infinity and DNE if the limit does not exist. You do not need to show work, but if you fail to show your work, you will not receive any partial credit for incorrect answers.

   (a) [3 points] $\lim_{x\to\infty} e^{-x}$

   (b) [3 points] $\lim_{x\to 0} \dfrac{\sin(x)}{x}$

(c) [3 points] $\lim_{x \to -2} \frac{x^2-4}{x+2}$

(d) [3 points] $\lim_{x \to \infty} x^2 \cdot \frac{x+1}{2x^3-4}$

(e) [3 points] $\lim_{x \to \infty} x^2 \cdot e^{-x^2}$

2. Calculate the derivative of each of the following functions. You do **not** need to revert to the limit definition, although if you find it helpful you may do so. Also, you do **not** need to simplify your answers.

   (a) [4 points] $f(x) = -\sin(\ln(x^2))$

   (b) [4 points] $f(x) = e^{\sin(x)}$

   (c) [4 points] $f(x) = \frac{x^2}{\sin(x)}$

(d) [4 points] $f(x) = 5\log_5(x)$

(e) [4 points] $f(x) = \tan^{-1}(4x + 1)$

3. Let $f(x) = \sqrt[3]{x}$.

(a) [3 points] What is the linearization of $f(x)$ at $x = 8$?

(b) [3 points] Use the linearization to estimate the value of $f(9)$.

4. Suppose $f(x)$ is a function.

   (a) [3 points] What is the slope of the [secant] line between the points $(a, f(a))$ and $(a+h, f(a+h))$?

   (b) [3 points] How can you use this expression to find the slope of the tangent line to the function $f(x)$ and the point $(a, f(a))$?

   (c) [9 points] In class we learned that your answer to part (b) was the definition of the derivative. Use this to find the derivative of $f(x) = \frac{1}{x}$.

5. (a) [8 points] Use implicit differentiation to find y'

$$1 + x^2 \tan(\pi \cdot y) + y^3 x = 2x$$

(b) [3 points] Use your work in part (a) to find the slope of the tangent line to the curve $1 + x^2 \tan(\pi \cdot y) + y^3 x = 2x$ at the point $(1, 1)$.

(c) [3 points] Now give the equation for the tangent line to the curve $1 + x^2 \tan(\pi \cdot y) + y^3 x = 2x$ at the point $(1, 1)$.

6. If you have ever driven around Elizabeth, New Jersey, you will have undoubtedly seen the large cylindrical storage tanks where gasoline is kept after it has been refined, but before it is shipped out to gas stations. Imagine that one of these cylindrical containers has a *height* of 30 meters, and a *diameter* of 30 meters, and that gasoline is being poured into the container at a rate of $1\frac{m^3}{sec}$. In this problem we will find the **rate of change of the depth** of the gasoline in the tank when the tank is half full. You may find it helpful to use the fact that the volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$.

   (a) [3 points] Draw a diagram and label it using notation for the quantities we are interested in.

   (b) [7 points] Solve the problem. In other words, find the rate of change of the depth of the gasoline in the tank when the tank is half full, and gasoline is being poured in at a rate of $1\frac{m^3}{sec}$.

7. Consider the function

$$f(x) = x^3 - 3x^2 - 9x + 7.$$

It will be used in the following four parts of this problem.

(a) [2 points] What are the domain and range of the function $f(x)$?

(b) [6 points] Find all local maximums, local minimums and inflection points of the function $f(x)$.

(c) [4 points] Sketch the graph of $f(x)$.

(d) [4 points] Use the Intermediate Value Theorem to prove that there is a root of the function $f(x)$ between $x = -3$ and $x = -1$. Recall that a root of a function is a number a in the domain of the function such that $f(a) = 0$.

8. In this problem, we will sketch the graph of

$$f(x) = \frac{x}{1 - x^2}$$

(a) [4 points] Identify all vertical and horizontal asymptotes of $f(x)$ and draw them on the graph in the space provided on the next page.

(b) [4 points] Take the derivative of f and use this to find any critical numbers of f.

(c) [4 points] Find the second derivative of f, and use this to find any inflection points of f.

(d) [2 points] Use the locations of the vertical asymptotes, the critical numbers and the inflection points to create intervals ($x < a$, $a < x < b$, ..., etc.), and determine whether f is increasing or decreasing and whether it is concave up or concave down on each of these intervals.

(e) [4 points] Use parts (a)-(d) to sketch the graph of f.

9. [8 points] Farmer John has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

10. (a) [2 points] What is Newton's method used for?

(b) [2 points] State the formula used in Newton's method for the second approximation, x_2, if x_1 is the initial approximation.

(c) [2 points] If I want to use Newton's Method to approximate $\sqrt{2}$, what function should I use?

11. Short answer

(a) [2 points] What is $\ln(e^5)$?

(b) [3 points] What is $\log_a(\sqrt{a})$.

(c) [3 points] What is the domain of the function $f(x) = \dfrac{x^2 - 1}{x - 1}$?

(d) [3 points] Is $f(x) = x^2 + x + 1$ even, odd or neither?

(e) [3 points] Solve for x: $\ln(x + 1) - \ln(5) = 3$

12. Label each statement as “true” or “false” (i.e., you must write the *whole* word). One point each.

    (a) [2 points] ____ $\log_a(X+Y) = \log_a(X) + \log_a(Y)$.

    (b) [2 points] ____ The function $f(x) = \sin(x)$ has an inflection point at $x = 0$.

    (c) [2 points] ____ If a particle’s location is given by $f(t) = \sin(5t)$, then it’s acceleration is given by $a(t) = -25\sin(t)$.

    (d) [2 points] ____ The graph of the function $10f(x)$ can be obtained by stretching $f(x)$ vertically by a factor of 10.