NAME: ____________________

MATH 1 MIDTERM 1

October 17, 2007

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- Print your name clearly in the space provided.
- You may not use a calculator.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	9	
2	12	
3	4	
4	6	
5	10	
6	10	
7	8	
8	10	
9	11	
10	20	
Total:	100	

1. Determine the inverse function $f^{-1}(x)$.

(a) [1 point] $f(x) = x^3$

$$f^{-1}(x) = \sqrt[3]{x}$$

(b) [1 point] $f(x) = 2^x$

$$f^{-1}(x) = \log_2 x$$

(c) [1 point] $f(x) = e^x$

$$f^{-1}(x) = \ln x$$

(d) [1 point] $f(x) = \log_3 x$

$$f^{-1}(x) = 3^x$$

(e) [1 point] $f(x) = \tan x$

$$f^{-1}(x) = \tan^{-1}(x) = \arctan(x)$$

(f) [2 points] $f(x) = -\frac{1}{x}$

$$y = \frac{-1}{x}$$

$$x = \frac{-1}{y} \quad \text{so} \quad f^{-1}(x) = \frac{-1}{x}$$

(g) [2 points] $f(x) = \sqrt{2x-1}$

$$y = \sqrt{2x-1}$$

$$x = \sqrt{2y-1}$$

$$x^2 = 2y-1 \longrightarrow x^2+1 = 2y$$

$$\frac{x^2+1}{2} = y \quad \text{so} \quad f^{-1}(x) = \frac{x^2+1}{2}$$

2. State the domain and range of the following functions.

(a) [2 points] $f(x) = x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(b) [2 points] $f(x) = \sqrt{x}$

Domain: $[0, \infty)$

Range: $[0, \infty)$

(c) [2 points] $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(d) [2 points] $f(x) = \sqrt[3]{x}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(e) [2 points] $f(x) = \frac{1}{x}$

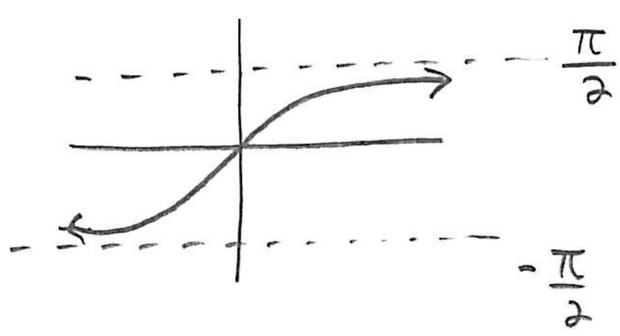
Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

(f) [2 points] $f(x) = \arctan x$

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



3. Let $f(x) = x^3 - 2x + 1$.

(a) [1 point] Compute $f(0)$.

$f(0) = 0^3 - 2(0) + 1 = 1$

(b) [1 point] Compute $f(2)$.

$f(2) = 2^3 - 2(2) + 1 = 8 - 4 + 1 = 5$

(c) [1 point] Find the slope of the line passing through the points $(0, f(0))$ and $(2, f(2))$.

$$m = \frac{f(0) - f(2)}{0 - 2} = \frac{1-5}{-2} = 2$$

(d) [1 point] Find the equation of the line passing through the points $(0, f(0))$ and $(2, f(2))$.

$y = mx + b$

$y = 2x + b$

$1 = 2(0) + b$

$1 = b$ $\therefore$ $y = 2x + 1$

4. Let $d(t) = t^2 - 1$ represent the distance an object has traveled in time t.

(a) [2 points] Determine the average velocity of the object in the interval $[1, 2]$.

$$\frac{d(2) - d(1)}{2-1} = \frac{(2^2-1) - (1^2-1)}{1} = 3$$

(b) [2 points] Evaluate and simplify $\frac{d(1+h)-d(1)}{h}$.

$$\frac{d(1+h) - d(1)}{h} = \frac{((1+h)^2 - 1) - 0}{h} = \frac{1+2h+h^2-1}{h}$$

$$= \frac{h(2+h)}{h}$$

$$= 2+h$$

(c) [2 points] The expression above represents the average velocity of the object in an interval $[1, 1+h]$. Plug in $h = 0.1, 0.01$, and 0.001 into the simplified form of the expression (or the complicated one if you prefer!) and estimate

$$\lim_{h \to 0} \frac{d(1+h) - d(1)}{h}.$$

$h = .1 \Rightarrow 2 + .1 = 2.1$

$h = .01 \Rightarrow 2 + .01 = 2.01$

$h = .001 \Rightarrow 2 + .001 = 2.001$

Estimate: $\lim_{h \to 0} \frac{d(1+h) - d(1)}{h} = 2$

5. Starting with the function $y = \frac{1}{x}$, obtain $f(x)$ by taking $\frac{1}{x}$ and translating it right one unit followed by reflecting it about the x-axis. Obtain $g(x)$ by taking $\frac{1}{x}$ and reflecting it about the x-axis followed by translating it up one unit.

(a) [2 points] What is $f(x)$?

$$-\left(\frac{1}{x-1}\right)$$

(b) [2 points] What is $g(x)$?

$$\frac{-1}{x}+1$$

(c) [2 points] Compute $f \circ g$.

$$f(g(x)) = f\left(\frac{-1}{x}+1\right) = -\left(\frac{1}{\left(\frac{-1}{x}+1\right)-1}\right) = -\left(\frac{1}{-\frac{1}{x}}\right) = x$$

(d) [2 points] Compute $g \circ f$.

$$g(f(x)) = g\left(-\left(\frac{1}{x-1}\right)\right) = \frac{-1}{\left(\frac{-1}{x-1}\right)} + 1 = x-1+1 = x$$

(e) [2 points] Given the results from parts (c) and (d), what relationship exists between f and g?

$g = f^{-1}$ or $f = g^{-1}$.

6. Simplify the following expressions:

(a) [2 points] $16^{-3/4}$

$$16^{-3/4} = \left(16^{1/4}\right)^{-3} = 2^{-3} = \frac{1}{8}$$

(b) [2 points] $\frac{x}{y} - \frac{y}{x}$

$$\frac{x}{y} - \frac{y}{x} = \frac{x^2}{xy} - \frac{y^2}{xy} = \frac{x^2-y^2}{xy}$$

(c) [2 points] $\log_8(64)$

$$\log_8(64) = \log_8(8^2) = 2\log_8(8) = 2$$

(d) [2 points] $\log_2(6) - \log_2(15) + \log_2(20)$

$$\log_2 6 - \log_2 15 + \log_2 20 = \log_2 \frac{6}{15} + \log_2 20$$

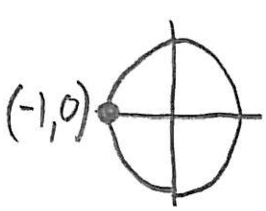
$$= \log_2\left(\frac{6}{15}\cdot 20\right) = \log_2\left(\frac{120}{15}\right)$$

$$= \log_2(8)$$

$$= \log_2(2^3)$$

$$= 3$$

(e) [2 points] $\arccos(-1)$



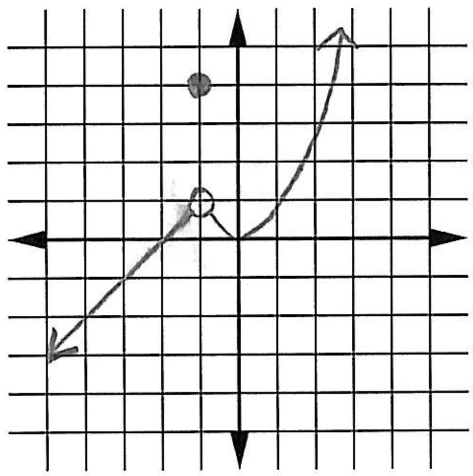
$$\cos(x) = -1$$

$$x = \pi$$

7. Let

$$f(x) = \begin{cases} x+2 & \text{if } x < -1 \\ x^2 & \text{if } x > -1 \\ 4 & \text{if } x = -1 \end{cases}.$$

(a) [4 points] Graph $f(x)$.



(b) [1 point] Find $\lim_{x\to -1^-} f(x)$.

1

(c) [1 point] Find $\lim_{x\to -1^+} f(x)$.

1

(d) [1 point] Find $\lim_{x\to -1} f(x)$.

1

(e) [1 point] Find $f(-1)$.

4

8. Solve for x.

(a) [2 points] $x - 3 = 2 - \frac{x}{2}$

$x - 3 = 2 - \frac{x}{2}$

$x + \frac{x}{2} = 2 + 3$

$1.5x = 5$

$x = \frac{5}{1.5} = \frac{10}{3}$

(b) [2 points] $\left(\frac{1}{3}\right)^x = 27$

$x = -3$

(c) [2 points] $\tan(x) = 1$ with x in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$

$x = \frac{\pi}{4}$ or $45°$ (when $\frac{\sin(x)}{\cos(x)} = 1$, so when $\sin(x) = \cos(x)$)

(d) [2 points] $\sin(\arcsin(x)) = 1$

1

(e) [2 points] $\ln((x+1)^3) = 3$

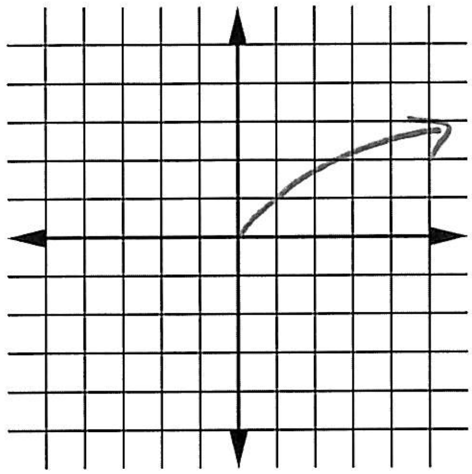
$\ln((x+1)^3) = 3$

$3\ln(x+1) = 3 \Rightarrow \ln(x+1) = 1$

$e^1 = x + 1$

$e - 1 = x$

9. [1 point] Sketch the graph of $f(x) = \sqrt{x}$.



Write the equations for the graphs that are obtained from the graph of $f(x)$ as follows:

(a) [2 points] Translate to the left by 3 units.

$\sqrt{x+3}$

(b) [2 points] Stretch horizontally by a factor of 4.

$\sqrt{\frac{x}{4}}$

(c) [2 points] Reflect about the y-axis.

$\sqrt{-x}$

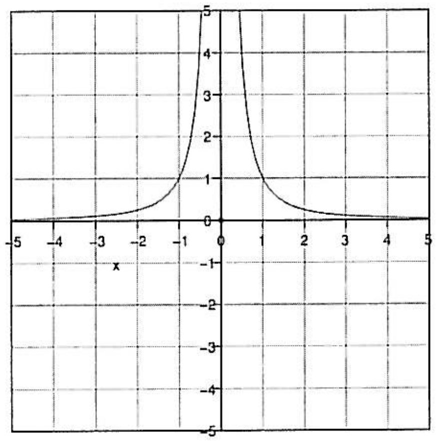
(d) [2 points] First (a) then (c).

$\sqrt{-x+3}$

(e) [2 points] First (b) then (a).

$\sqrt{\frac{x+3}{4}}$

10. Consider the function $f(x) = \frac{1}{x^2}$ graphed below.



(a) [2 points] Find the domain of f.

$(-\infty, 0) \cup (0, \infty)$

(b) [2 points] Find the range of f.

$X > 0$

(c) [1 point] Is f one-to-one?

No

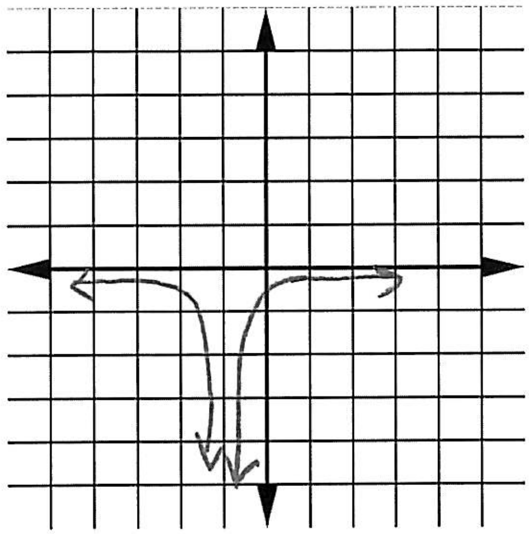
(d) [1 point] What kind of symmetry does f have (even, odd, neither)?

Even

(e) [2 points] Determine the two transformations (in order) needed to obtain $g(x) = -\frac{1}{(x+1)^2}$ from $f(x)$.

Shift 1 unit left, reflect across x-axis
(order does not matter)

(f) [1 point] Sketch the graph of $g(x)$.



(g) [2 points] Find the domain of g.

$(-\infty, -1) \cup (-1, \infty)$

(h) [2 points] Find the range of g.

$X < 0$

(i) [1 point] Find $\lim_{x \to 0^+} g(x)$.

-1

(j) [1 point] Find $\lim_{x \to 0^-} g(x)$.

-1

(k) [1 point] Find $\lim_{x \to 0} g(x)$.

-1

(l) [1 point] Find $g(0)$. -1

(m) [1 point] Find $\lim_{x \to -1^+} g(x)$. $-\infty$

(n) [1 point] Find $\lim_{x \to -1^-} g(x)$. $-\infty$

(o) [1 point] Find $\lim_{x \to -1} g(x)$.

$-\infty$