Find the exponential function $f(x) = Ca^x$ whose graph is given.

Solution, method 1. We need to find both C and a, using the two points that are given on the graph, namely $(0, 2)$ and $(2, \frac{2}{9})$. To do this, we plug the two points into the equation for $f(x)$. If we plug in the first point, we get

$$2 = Ca^0.$$

Since anything to the power 0 is 1, the equation becomes

$$2 = C \cdot 1,$$

so $C = 2$. Now we can plug in the second point, along with the value of C that we determined:

$$\frac{2}{9} = 2a^2.$$

If we divide both sides by 2, we get

$$\frac{1}{9} = a^2.$$

Finally, we take the square root of both sides:

$$\pm\frac{1}{3} = a.$$

Do both values of a work? Remembering that a is the base of our exponential, and knowing that bases have to be positive, we can single out the positive value as our answer. Thus $C = 2$ and $a = \frac{1}{3}$.

Solution, method 2. We notice that $f(x) = Ca^x$ looks like a transformation of the basic exponential function $g(x) = a^x$, with the exponential stretched vertically by a factor of C. We know that the original exponential had to cross the y-axis at $y = 1$, but looking at the graph, we see that it crosses the y-axis at $y = 2$. Thus the stretching factor C must be 2. If we undo the stretch in the graph, the second point, $(2, \frac{2}{9})$, would shrink to $(2, \frac{1}{9})$. Plugging this into $g(x)$, we get

$$\frac{1}{9} = a^2$$

as in method 1, so we again get $a = \frac{1}{3}$.

Solution, method 3. Here's a trick that will work when we don't have the y-intercept. We plug the two points into the equation to get two equations,

$$2 = Ca^0 \quad \text{and} \quad \frac{2}{9} = Ca^2.$$

Dividing the second equation by the first gives us

$$\frac{2/9}{2} = \frac{Ca^2}{Ca^0},$$

which after canceling each side becomes

$$\frac{1}{9} = a^2.$$

As we saw before, this means that $a = \frac{1}{3}$. Now that we know a, we can plug it into one of the original equations, say $\frac{2}{9} = Ca^2$. This gives us

$$\frac{2}{9} = C \cdot \frac{1}{9}.$$

Multiplying both sides by 9, we have $C = 2$.