Name: KEY

Math 1 Fall 2005 Exam 1

Instructions

This exam is being given under the Dartmouth College Honor Principle. It is a closed book, closed note exam and calculators are not permitted. All work is to be your own. If work is not shown on problems where we view it as necessary, we reserve the right to give you no credit. Do not use formulas you may know for derivatives on this exam. Please box your answers. Good luck!

Show your work.

For questions 1-2, refer to the circle that has a diameter with endpoints at $(-2, 4)$ and $(10, -2)$.

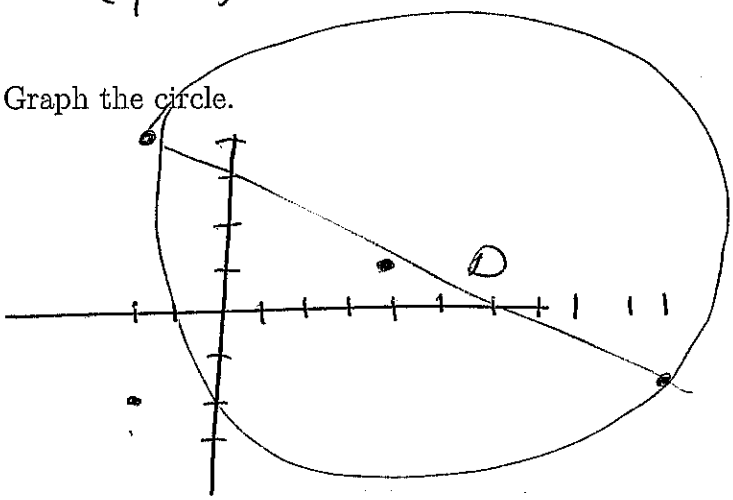
1. (5 points) Find the equation of this circle.

center: $(4, 1)$

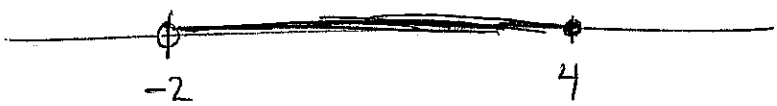
radius: $\sqrt{6^2 + 3^2} = \sqrt{45}$

$(x-4)^2 + (y-1)^2 = 45$

2. (5 points) Graph the circle.



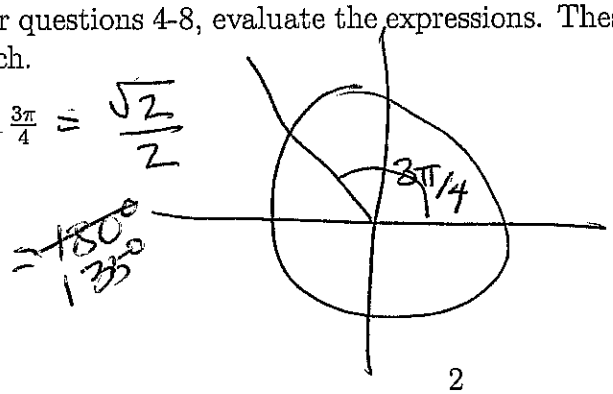
3. (5 points) Sketch the set of every real number x so that $x > -2$ and $x \leq 4$. Write this set as an interval.



$(-2, 4]$

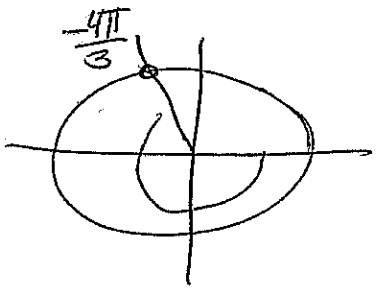
For questions 4-8, evaluate the expressions. These questions are worth 3 points each.

4. $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$



$= 180° - 135°$

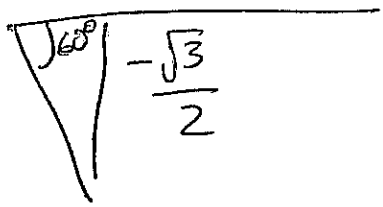
5. $\cos\frac{-4\pi}{3} = -\frac{1}{2}$



6. $\sin^{-1}\frac{-\sqrt{3}}{2}$

$-\frac{\pi}{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

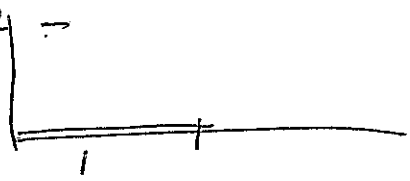


7. $\tan(\cos^{-1} 1)$

$\cos\theta = 1 \Rightarrow \theta = 0$

$\tan(0) = \frac{\sin 0}{\cos 0} = \frac{0}{1}$

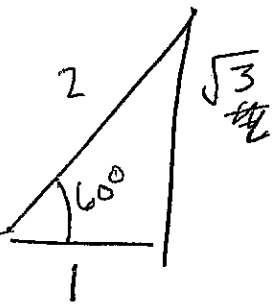
$= \boxed{0}$



8. $\sin(\tan^{-1}\sqrt{3})$.

$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

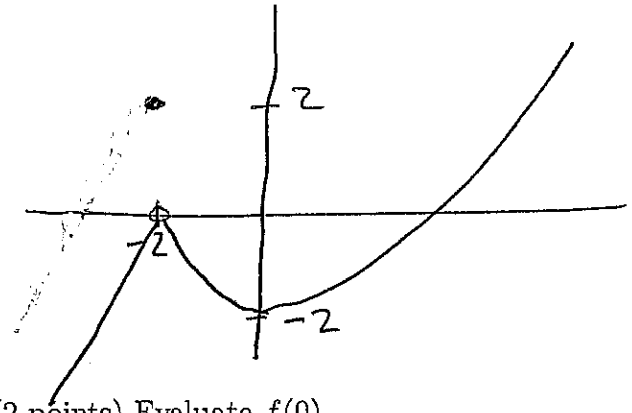
$\sin\frac{\pi}{3} = \boxed{\frac{\sqrt{3}}{2}}$



For questions 9-16 refer to the function

$$f(x) = \begin{cases} 3x+6 & x < -2 \\ \frac{1}{2}x^2 - 2 & x > -2 \\ 2 & x = -2 \end{cases}$$

9. (2 points) Graph this function.



10. (2 points) Evaluate $f(0)$.

$f(0) = -2$

11. (2 points) Evaluate $f(-2)$.

$f(-2) = 2$

12. (2 points) Evaluate $f(-3)$

$f(-3) = 3(-3)+6 = -3$

13. (2 points) Evaluate $\lim_{x \to -2^-} f(x)$ $= 0$

14. (2 points) Evaluate $\lim_{x \to -2^+} f(x)$ $= 0$

15. (2 points) Evaluate $\lim_{x \to -2} f(x)$ $= 0$

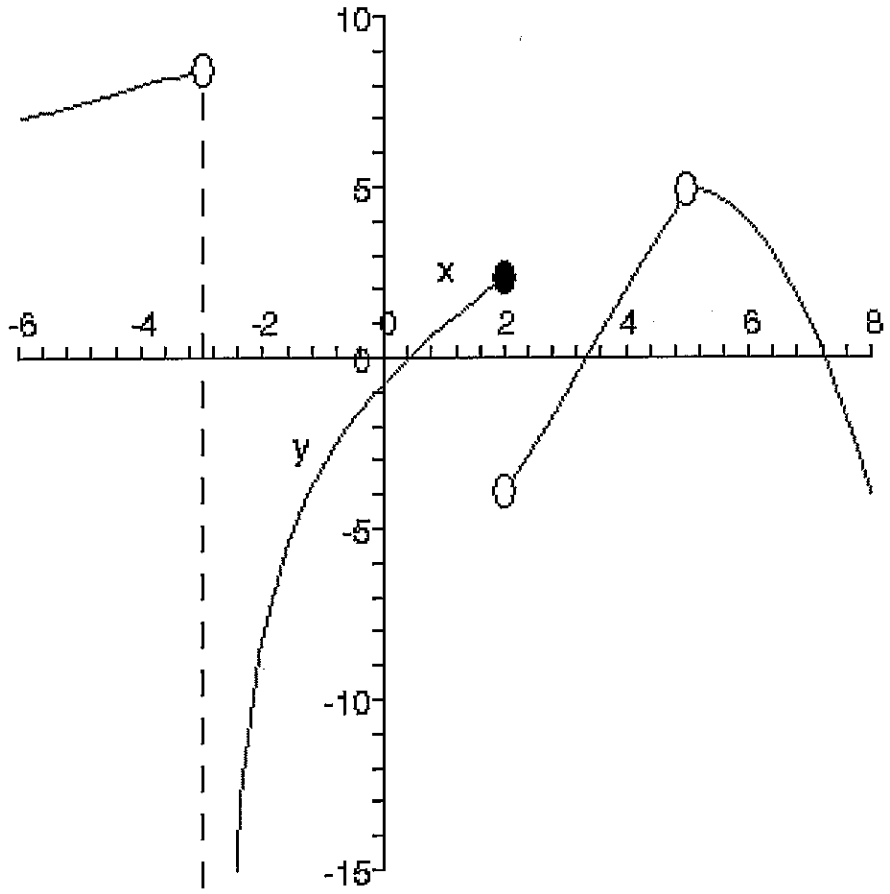
16. (6 points) Is $f(x)$ continuous at $x = -2$? Why or why not?

NO : $f(-2) \neq \lim_{x \to -2} f(x)$.

17. (5 points) What is the slope of the line through the points $(2, -1)$ and $(-3, 2)$?

$m = \frac{2 - -1}{-3 - 2} = \boxed{\frac{3}{-5}}$

For questions 18-22, refer to the following graph of $g(x)$. Your answers may be rounded to the nearest integer.



18. (2 points) Evaluate $\lim_{x \to -3^+} g(x)$

-∞

19. (2 points) Evaluate $\lim_{x \to 2^+} g(x)$

-4

20. (2 points) Evaluate $\lim_{x \to 5^+} g(x)$

5

21. (2 points) Evaluate $\lim_{x \to -3} g(x)$

DNE

22. (2 points) Evaluate $\lim_{x \to 5} g(x)$

5

For Questions 23-27, evaluate the limit and justify your answer with a sentence or two.

23. (3 points) $\lim_{x \to 2} \frac{x-2}{x^3-2x^2+x-2}$ $= \lim_{x \to 2} \frac{x-2}{(x-2)(x^2+1)} = \lim_{x \to 2} \frac{1}{x^2+1}$

$= \frac{1}{4+1} = \boxed{1/5}$

$\frac{1}{x^2+1}$ is a cts. function equal to $\frac{x-2}{x^3-2x^2+x-2}$ except at $x = 2$.

24. (3 points) $\lim_{x \to 5^-} \frac{49}{(x-5)^3}$ $= -\infty$

$(x-5)^3$ is negative for $x < 5$, and $x = 5$ is a vertical assymptote for this function.

25. (3 points) $\lim_{x\to\infty} \frac{2x^2-3x+1}{4x^2+3x-2}$ $= \lim_{x\to\infty} \frac{2-\frac{3}{x}-\frac{1}{x^2}}{4+\frac{3}{x}-\frac{2}{x^2}} = \frac{\lim_{x\to\infty} 2-\frac{3}{x}-\frac{1}{x^2}}{\lim_{x\to\infty} 4+\frac{3}{x}-\frac{2}{x^2}}$

$= \frac{2}{4} = \frac{1}{2}$

The functions are equal, except at $x=0$, and the limit of the quotient is the quotient of the limit.

26. (3 points) $\lim_{x\to-\infty} 3-\frac{2}{x}+1$ $= 4$

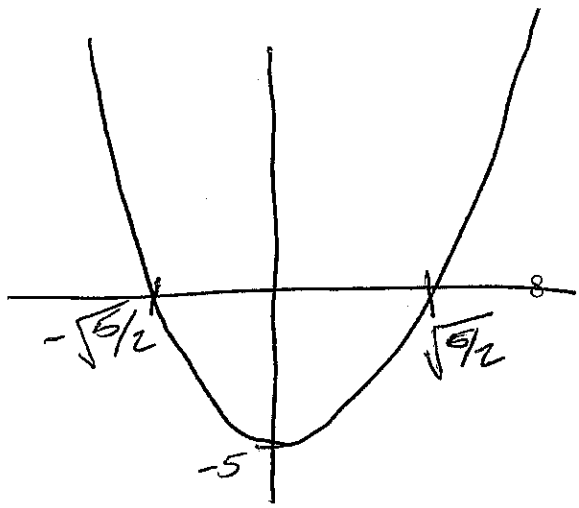
The limit of the sum is the sum of the limits, and $\lim_{x\to-\infty} \frac{-2}{x} = 0$

27. (3 points) $\lim_{x\to 5} \sqrt{x+1}-x^2$ $= \sqrt{5+1} - 5^2 = \sqrt{6} - 25$

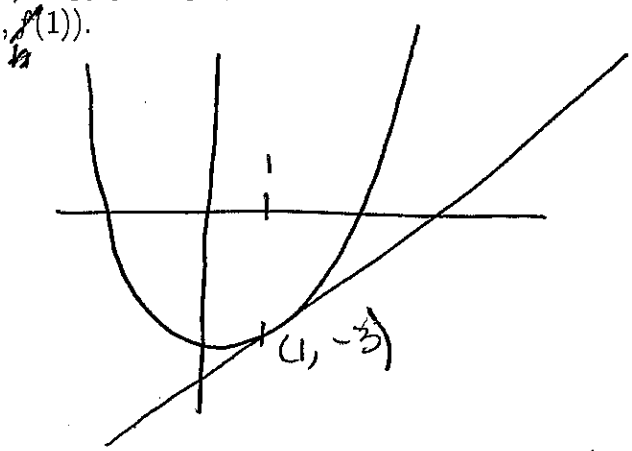
This is a cts function at $x = 5$.

For questions 28-32 refer to the function $f(x) = 2x^2 - 5$.

28. (2 points) Graph this function.



29. (2 points) Copy your graph from question 28 and graph the tangent line at the point $(1, f(1))$.



30. (2 points) What is the slope of the tangent line at $(1, f(1))$?

$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{2(1+h)^2 - 5 - (2(1)^2 - 5)}{h}$$

$$= \lim_{h\to 0} \frac{2(1+2h+h^2) - 2}{h} = \lim_{h\to 0} \frac{2+4h+2h^2-2}{h}$$

$$= \lim_{h\to 0} 4+2h = \boxed{4}$$

31. (2 points) What is the equation of the tangent line at $(1, f(1))$?

$$y + 3 = 4(x - 1)$$

$$y = 4x - 7$$

32. (2 points) What is the slope of the tangent line at a point with x-coordinate 0?

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{2(0+h)^2 - 5 - (2(0)^2 - 5)}{h}$$

$$= \lim_{h\to 0} \frac{2h^2}{h} = \lim_{h\to 0} 2h = \boxed{0}$$

33. (10 points) For the function $f(x) = -x^2 + 3x - 1/4$, evaluate

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\left(-(a+h)^2 + 3(a+h) - 1/4\right) - \left(-a^2+3a-\frac{1}{4}\right)}{h}$$

$$= \lim_{h \to 0} \frac{-(a^2+2ah+h^2) + 3a + 3h - 1/4 + a^2 - 3a + 1/4}{h}$$

$$= \lim_{h \to 0} \frac{-2ah - h^2 + 3h}{h}$$

$$= \lim_{h \to 0} -2a - h + 3$$

$$= -2a + 3$$