1. Indicate whether each of the following statements is true or false.

   (a) If f is continuous at a, then f is differentiable at a.

   (b) If f and g are differentiable, then
   $$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x).$$

   (c) If f and g are differentiable, then
   $$\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x).$$

   (d) If f and g are differentiable, then
   $$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

   (e) If f is differentiable, then
   $$\frac{d}{dx}[\sqrt{f(x)}] = \frac{f'(x)}{2\sqrt{f(x)}}.$$

   (f) If f is differentiable, then
   $$\frac{d}{dx}[f(\sqrt{x})] = \frac{f'(x)}{2\sqrt{x}}.$$

   (g) An equation of the tangent line to the parabola $y = x^2$ at $(-2, 4)$ is $y - 4 = 2x(x+2)$.

2. Find the derivative of the following functions:

$k(x) = \sqrt{x^2+7x}$	$h(t) = (t - \frac{1}{t})^{\frac{3}{2}}$
$F(s) = \sqrt{s^3+1}(s^2+1)^4$	$F(y) = (\frac{y-6}{y+7})^3$
$s(t) = \sqrt[4]{\frac{t^3+1}{t^3-1}}$	$f(z) = \frac{1}{\sqrt[5]{2z-1}}$
$y = \sqrt{x+\sqrt{x+\sqrt{x}}}$	$f(x) = \frac{\sqrt{1-x^2}}{x}$
$y = \ln(x^2e^x)$	$f(x) = e^{\sqrt{x}}$
$f(x) = xe^{-x^2}$	$y = xe^{2x}$
$h(t) = \sqrt{1-e^t}$	$k(x) = \sqrt[3]{1+\sqrt{x}}$

$y = e^{-\frac{1}{x}}$

$y = \frac{e^{-x^2}}{x}$

$y = e^{x+e^x}$

$y = \sqrt[3]{2x + e^{3x}}$

$f(x) = \ln(x+1)$

$f(x) = x^2 \ln(1-x^2)$

$f(x) = \ln(\ln(\ln x))$

$f(x) = \sqrt{x} \ln x$

$g(x) = \ln(\frac{a-x}{a+x})$

$h(x) = \ln(x + \sqrt{x^2 - 1})$

$F(x) = \ln(\sqrt{x})$

$G(x) = \sqrt{\ln x}$

$g(u) = \frac{1-\ln u}{1+\ln u}$

$G(u) = \ln(\sqrt{\frac{3u+2}{3u-2}})$

$y = \ln(x + \ln x)$

$F(x) = e^x \ln x$

3. For each of the following functions, find y' and y''.

   (a) $y = x \ln x$
   (b) $y = \ln(ax)$

4. For each of the following, find the equation of the tangent line to the given curve at the given point.

   (a) $y = (x^3 - x^2 + x - 1)^{10}$, $(1, 0)$
   (b) $y = \sqrt{x + \frac{1}{x}}$, $(1, \sqrt{2})$
   (c) $y = \frac{8}{\sqrt{4+3x}}$, $(4, 2)$
   (d) $y = x^2 e^{-x}$, $(1, \frac{1}{e})$
   (e) $y = x \ln x$, (e, e)
   (f) $y = \ln(\ln x)$, $(e, 0)$

5. Find an equation of the tangent to the curve $y = e^{-x}$ that is perpendicular to the line $2x - y = 8$.

6. Find the points at which the tangent line to the curve $y = [\ln(x+4)]^2$ is horizontal.

7. For each of the following, find f' in terms of g'.

   (a) $f(x) = [g(x)]^2$
   (b) $f(x) = g(g(x))$
   (c) $f(x) = g(x^2)$
   (d) $f(x) = x^a g(x^b)$

(e) $f(x) = g(e^x)$

(f) $f(x) = g(\ln x)$

(g) $f(x) = \ln(g(e^x))$

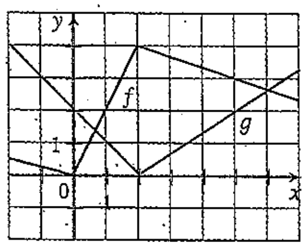
8. Find h' in terms of f' and g' if $h(x) = \sqrt{\frac{f(x)}{g(x)}}$.

9. The function g is a twice differentiable function. For each of the following functions, find f'' in terms of g, g', and g''.

(a) $f(x) = xg(x^2)$

(b) $f(x) = g(\sqrt{x})$

10. If f and g are functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$. Find each of the following derivatives, if it exists.



(a) $u'(1)$

(b) $v'(1)$

(c) $w'(1)$

11. Find the following limits.

(a) $\lim_{x\to 1} \frac{\ln x}{x-1}$

(b) $\lim_{x\to \infty} \frac{\ln x}{x}$

(c) $\lim_{x\to 0+} \frac{\ln x}{\sqrt{x}}$

(d) $\lim_{x\to \infty} \frac{(\ln x)^3}{x^2}$

(e) $\lim_{x\to 2^-} \frac{\ln x}{\sqrt{2-x}}$

(f) $\lim_{x\to\infty} \frac{\ln(\ln x)}{\sqrt{x}}$

(g) $\lim_{x\to\infty} \frac{\ln(1+e^x)}{5x}$

(h) $\lim_{x\to 0^+} \sqrt{x}\ln x$

(i) $\lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$

12. For each of the following functions find

- the intervals of increase or decrease,
- the local maximum or minimum values,
- the intervals of concavity, and
- the x-coordinates of the points of inflection.

(a) $h(x) = (x^2-1)^3$

(b) $P(x) = x\sqrt{x^2+1}$

(c) $P(x) = x\sqrt{x+1}$

(d) $Q(x) = x^{\frac{1}{3}}(x+3)^{\frac{2}{3}}$

1) (a) F

(b) T

(c) F

(d) T

(e) T

(f) F

(g) F

2)

• $\frac{d}{dx}\left[\sqrt{x^2+7x}\right]$

$= \frac{1}{2}(x^2+7x)^{-\frac{1}{2}}(2x+7)$

$= \underline{\underline{\dfrac{2x+7}{2\sqrt{x^2+7x}}}}$

• $\frac{d}{dt}\left[(t-\frac{1}{t})^{\frac{3}{2}}\right]$

$= \underline{\underline{\frac{3}{2}(t-\frac{1}{t})^{\frac{1}{2}}(1+\frac{1}{t^2})}}$

• $\frac{d}{ds}\left[\sqrt{s^3+1}\,(s^2+1)^4\right]$

$= \frac{1}{2}(s^3+1)^{-\frac{1}{2}}(3s^2)(s^2+1)^4$

$+ \sqrt{s^3+1}\; 4(s^2+1)^3(2s)$

$= \frac{3}{2}s^2(s^3+1)^{-\frac{1}{2}}(s^2+1)^4$

$+ 8s\sqrt{s^3+1}\,(s^2+1)^3$

$= \frac{1}{2}s(s^3+1)^{-\frac{1}{2}}(s^2+1)^3\left[3s(s^2+1)+16(s^3+1)\right]$

$= \underline{\underline{\frac{1}{2}s(s^3+1)^{-\frac{1}{2}}(s^2+1)^3(19s^3+3s+16)}}$

• $\frac{d}{dy}\left[\left(\frac{y-6}{y+7}\right)^3\right]$

$= 3\left(\frac{y-6}{y+7}\right)^2 \frac{y+7-(y-6)}{(y+7)^2}$

$= \underline{\underline{\dfrac{39(y-6)^2}{(y+7)^4}}}$

• $\frac{d}{dt}\left[\sqrt[4]{\frac{t^3+1}{t^3-1}}\right]$

$= \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-\frac{3}{4}} \frac{3t^2(t^3-1)-3t^2(t^3+1)}{(t^3-1)^2}$

$= \underline{\underline{-\frac{3t^2}{2(t^3-1)^2}\left(\frac{t^3+1}{t^3-1}\right)^{-\frac{3}{4}}}}$

• $\frac{d}{dz}\left[\frac{1}{\sqrt[5]{2z-1}}\right]$

$= -\frac{1}{5}(2z-1)^{-\frac{6}{5}}\cdot 2$

$= \underline{\underline{-\frac{2}{5}(2z-1)^{-\frac{6}{5}}}}$

• $\frac{d}{dx}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}\right]$

$= \frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-\frac{1}{2}}\left(1+\frac{1}{2}(x+\sqrt{x})^{-\frac{1}{2}}(1+\frac{1}{2}x^{-\frac{1}{2}})\right)$

$= \underline{\underline{\dfrac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\dfrac{1}{2\sqrt{x}}\right)\right)}}$

• $\frac{d}{dx}\left[\frac{\sqrt{1-x^2}}{x}\right] = \frac{d}{dx}\left[x^{-1}\sqrt{1-x^2}\right]$

$= x^{-1}\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) - x^{-2}\sqrt{1-x^2}$

$= -(1-x^2)^{-\frac{1}{2}} - x^{-2}\sqrt{1-x^2}$

$= (1-x^2)^{-\frac{1}{2}}\left(-1-\frac{1}{x^2}(1-x^2)\right)$

$= \underline{\underline{-\dfrac{1}{x^2\sqrt{1-x^2}}}}$

• $\frac{d}{dx}\left[\ln(x^2e^x)\right]$

$= \frac{1}{x^2e^x}(x^2e^x+2xe^x)$

$= \frac{(2+x)\cancel{e^x}}{x\cancel{e^x}} = \underline{\underline{\dfrac{2+x}{x}}}$

• $\frac{d}{dx}\left[e^{\sqrt{x}}\right]$

$= e^{\sqrt{x}}\,\frac{1}{2}x^{-\frac{1}{2}} = \underline{\underline{\frac{1}{2\sqrt{x}}e^{\sqrt{x}}}}$

• $\frac{d}{dx}\left[xe^{-x^2}\right]$

$= e^{-x^2} + xe^{-x^2}(-2x)$

$= \underline{\underline{(1-2x^2)e^{-x^2}}}$

• $\frac{d}{dx}\left[xe^{2x}\right]$

$= e^{2x} + xe^{2x}\cdot 2 = \underline{\underline{(1+2x)e^{2x}}}$

• $\frac{d}{dt}\left[\sqrt{1-e^t}\right] = \frac{1}{2}(1-e^t)^{-\frac{1}{2}}(-e^t)$

$= \underline{\underline{-\dfrac{e^t}{2\sqrt{1-e^t}}}}$

• $\frac{d}{dx}\left[\sqrt[3]{1+\sqrt{x}}\right]$

$= \frac{1}{3}(1+\sqrt{x})^{-\frac{2}{3}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$= \underline{\underline{\frac{1}{6\sqrt{x}}(1+\sqrt{x})^{-\frac{2}{3}}}}$

- $\frac{d}{dx}\left[e^{-\frac{1}{x}}\right]$

$$= e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)$$
$$= \underline{\underline{\frac{1}{x^2}\, e^{-\frac{1}{x}}}}$$

- $\frac{d}{dx}\left[\frac{e^{-x^2}}{x}\right]$

$$= \frac{d}{dx}\left[x^{-1}e^{-x^2}\right]$$
$$= x^{-1}e^{-x^2}(-2x) + (-x^{-2})\,e^{-x^2}$$
$$= -2\, e^{-x^2} - x^{-2}e^{-x^2}$$
$$= \underline{\underline{-x^{-2}e^{-x^2}(2x^2+1)}}$$

- $\frac{d}{dx}\left[e^{x+e^x}\right]$

$$= \underline{\underline{e^{x+e^x}(1+e^x)}}$$

- $\frac{d}{dx}\left[\sqrt[3]{2x+e^{3x}}\right]$

$$= \frac{1}{3}(2x+e^{3x})^{-\frac{2}{3}}(2+3e^{3x})$$
$$= \underline{\underline{\left(\frac{2}{3}+e^{3x}\right)(2x+e^{3x})^{-\frac{2}{3}}}}$$

- $\frac{d}{dx}\left[\ln(x+1)\right] = \underline{\underline{\frac{1}{x+1}}}$

- $\frac{d}{dx}\left[x^2\ln(1-x^2)\right]$

$$= 2x\ln(1-x^2) + x^2\frac{1}{1-x^2}(-2x)$$
$$= \underline{\underline{2x\left(\ln(1-x^2) - \frac{x^2}{1-x^2}\right)}}$$

- $\frac{d}{dx}\left[\ln(\ln(\ln x))\right]$

$$= \frac{1}{\ln(\ln x)}\cdot\frac{1}{\ln x}\cdot\frac{1}{x}$$
$$= \underline{\underline{\frac{1}{x\ln x\cdot\ln(\ln x)}}}$$

- $\frac{d}{dx}\left[\sqrt{x}\,\ln x\right]$

$$= \frac{1}{2\sqrt{x}}\ln x + \frac{\sqrt{x}}{x}$$
$$= \underline{\underline{\frac{1}{2\sqrt{x}}(\ln x + 2)}}$$

- $\frac{d}{dx}\left[\ln\left(\frac{a-x}{a+x}\right)\right]$

$$= \frac{a+x}{a-x}\cdot\frac{-(a+x)-(a-x)}{(a+x)^2}$$
$$= \underline{\underline{\frac{2a}{(x-a)(a+x)}}}$$

- $\frac{d}{dx}\left[\ln(x+\sqrt{x^2-1})\right]$

$$= \frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}}\cancel{2}x\right)$$
$$= \underline{\underline{\frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{x}{\sqrt{x^2-1}}\right)}}$$

- $\frac{d}{dx}\left[\ln(\sqrt{x})\right]$

$$= \frac{1}{\sqrt{x}}\cdot\frac{1}{2}x^{-\frac{1}{2}} = \underline{\underline{\frac{1}{2x}}}$$

- $\frac{d}{dx}\left[\sqrt{\ln x}\right]$

$$= \frac{1}{2}(\ln x)^{-\frac{1}{2}}\left(\frac{1}{x}\right) = \underline{\underline{\frac{1}{2x\sqrt{\ln x}}}}$$

- $\frac{d}{du}\left[\frac{1-\ln u}{1+\ln u}\right]$

$$= \frac{-\frac{1}{u}(1+\ln u) - \frac{1}{u}(1-\ln u)}{(1+\ln u)^2}$$
$$= \underline{\underline{\frac{-2}{u(1+\ln u)^2}}}$$

- $\frac{d}{du}\left[\ln\left(\sqrt{\frac{3u+2}{3u-2}}\right)\right]$

$$= \sqrt{\frac{3u-2}{3u+2}}\left(\frac{1}{2}\left(\frac{3u+2}{3u-2}\right)^{-\frac{1}{2}}\frac{3(3u-2)-3(3u+2)}{(3u-2)^2}\right)$$
$$= -\frac{6}{(3u-2)^2}\left(\frac{3u-2}{3u+2}\right)$$
$$= \underline{\underline{\frac{-6}{(3u-2)(3u+2)}}}$$

- $\frac{d}{dx}\left[\ln(x+\ln x)\right]$

$$= \frac{1}{x+\ln x}\left(1+\frac{1}{x}\right)$$
$$= \underline{\underline{\frac{x+1}{x(x+\ln x)}}}$$

- $\frac{d}{dx}\left[e^x\ln x\right]$

$$= e^x\frac{1}{x} + e^x\ln x$$
$$= \underline{\underline{e^x\left(\ln x + \frac{1}{x}\right)}}$$

3) (a) $y = x \ln x$

$y' = \ln x + x\left(\frac{1}{x}\right) = \underline{\underline{1 + \ln x}}$

$y'' = \underline{\underline{\frac{1}{x}}}$

(b) $y = \ln(ax)$

$y' = \frac{a}{ax} = \underline{\underline{\frac{1}{x}}}$

$y'' = \underline{\underline{-\frac{1}{x^2}}}$

4) (a) $y = (x^3 - x^2 + x - 1)^{10}$, $(1, 0)$

$y' = 10(x^3 - x^2 + x - 1)^9(3x^2 - 2x + 1)$

$y'(1) = 10(1 - 1 + 1 - 1)^9(3 - 2 + 1)$

$= 0$

$\underline{\underline{y = 0}}$

(b) $y = \sqrt{x + \frac{1}{x}}$, $(1, \sqrt{2})$

$y' = \frac{1}{2}\left(x + \frac{1}{x}\right)^{-\frac{1}{2}}\left(1 - \frac{1}{x^2}\right)$

$y'(1) = \frac{1}{2}(1+1)^{-\frac{1}{2}}(1 - 1) = 0$

$\underline{\underline{y = \sqrt{2}}}$

(c) $y = \frac{8}{\sqrt{4 + 3x}}$, $(4, 2)$

$y' = -\frac{1}{2}(4 + 3x)^{-\frac{3}{2}} \cdot 3 \cdot 8$

$= -12(4 + 3x)^{-\frac{3}{2}}$

$y'(4) = -12(16)^{-\frac{3}{2}}$

$= -12(64)^{-1}$

$= -\frac{3}{16}$

$y = -\frac{3}{16}x + b$

$2 = -\frac{3}{16}(4) + b$

$= -\frac{3}{4} + b$

$\frac{11}{4} = b$

$\underline{\underline{y = -\frac{3}{16}x + \frac{11}{4}}}$

(d) $y = x^2 e^{-x}$, $\left(1, \frac{1}{e}\right)$

$y' = 2xe^{-x} - x^2 e^{-x} = (2 - x)xe^{-x}$

$y'(1) = e^{-1}$

$y = \frac{1}{e}x + b$

$\frac{1}{e} = \frac{1}{e} + b$

$0 = b$

$\underline{\underline{y = \frac{1}{e}x}}$

(e) $y = x \ln x$, (e, e)

$y' = \ln x + x\left(\frac{1}{x}\right) = 1 + \ln x.$

$y'(e) = 2$

$y = 2x + b$

$e = 2e + b$

$-e = b$

$\underline{\underline{y = 2x - e}}$

(f) $y = \ln(\ln x)$, $(e, 0)$

$y' = \frac{1}{\ln x} \; \frac{1}{x} = \frac{1}{x \ln x}$

$y'(e) = \frac{1}{e}$

$y = \frac{1}{e}x + b$

$0 = 1 + b$

$-1 = b$

$\underline{\underline{y = \frac{1}{e}x - 1}}$

5) $y = e^{-x}$

$y' = -e^{-x}$

slope of $2x - y = 8$ is 2

$-e^{-x} = -\frac{1}{2}$

$e^{-x} = \frac{1}{2}$

$-x = \ln\left(\frac{1}{2}\right) = -\ln(2)$

$x = \ln(2)$ $\quad e^{-\ln 2} = e^{\ln \frac{1}{2}} = \frac{1}{2}$

$y = -\frac{1}{2}x + b$

$\frac{1}{2} = -\frac{1}{2}\ln(2) + b$

$\frac{1}{2}(1 + \ln(2)) = b$

$\underline{\underline{y = -\frac{1}{2}x + \frac{1}{2}(1 + \ln 2)}}$

6) $y = [\ln(x+4)]^2$

$y' = 2\ln(x+4)\,\frac{1}{x+4}$

$= \frac{2}{x+4}\ln(x+4)$

$y' = 0$

$\frac{2}{x+4}\ln(x+4) = 0$

$\ln(x+4) = 0$

$x+4 = 1$

$x = -3$

$\underline{\underline{(-3, 0)}}$

7) (a) $f(x) = [g(x)]^2$

$f'(x) = \underline{\underline{2g(x)\,g'(x)}}$

(b) $f(x) = g(g(x))$

$f'(x) = \underline{\underline{g'(g(x))\,g'(x)}}$

(c) $f(x) = g(x^2)$

$f'(x) = g'(x^2)\cdot 2x$

$= \underline{\underline{2x\,g'(x^2)}}$

(d) $f(x) = x^a g(x^b)$

$= ax^{a-1}g(x^b) + x^a g'(x^b)bx^{b-1}$

$= \underline{\underline{ax^{a-1}g(x^b) + bx^{a+b-1}g'(x^b)}}$

(e) $f(x) = g(e^x)$

$f'(x) = \underline{\underline{g'(e^x)\,e^x}}$

(f) $f(x) = g(\ln x)$

$f'(x) = g'(\ln x)\frac{1}{x} = \underline{\underline{\frac{g'(\ln x)}{x}}}$

(g) $f(x) = \ln(g(e^x))$

$f'(x) = \frac{1}{g(e^x)}\,g'(e^x)\,e^x$

$= \underline{\underline{\frac{e^x g'(e^x)}{g(e^x)}}}$

8) $h(x) = \sqrt{\frac{f(x)}{g(x)}}$

$h'(x) = \underline{\underline{\frac{1}{2}\left(\frac{f(x)}{g(x)}\right)^{-\frac{1}{2}}\ \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}}}$

9) (a) $f(x) = x\,g(x^2)$

$f'(x) = g(x^2) + x\,g'(x^2)\,2x$

$= g(x^2) + 2x^2 g'(x^2)$

$f''(x) = g'(x^2)2x + 2x^2 g''(x^2)2x + 4x\,g'(x^2)$

$= \underline{\underline{6x\,g'(x^2) + 4x^3 g''(x^2)}}$

(b) $f(x) = g(\sqrt{x})$

$f'(x) = g'(\sqrt{x})\,\frac{1}{2\sqrt{x}}$

$f''(x) = g''(\sqrt{x})\frac{1}{2\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) + g'(\sqrt{x})\,\frac{1}{2}\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= \underline{\underline{\frac{1}{4x}g''(\sqrt{x}) - \frac{1}{4}x^{-\frac{3}{2}}g'(\sqrt{x})}}$

10) $u(x) = f(g(x))$

$v(x) = g(f(x))$

$w(x) = g(g(x))$

(a) $u'(x) = f'(g(x))\,g'(x)$

$u'(1) = f'(g(1))\,g'(1)$

$= f'(1)\,g'(1)$

$= -1(2) = \underline{\underline{-2}}$

(b) $v'(x) = g'(f(x))\,f'(x)$

$v'(1) = g'(f(1))\,f'(1)$

$= g'(2)\,f'(1)$

$\underline{\underline{\text{does not exist}}}$

(c) $w'(x) = g'(g(x))\,g'(x)$

$w'(1) = g'(g(1))\,g'(1)$

$= g'(1)\,g'(1)$

$= (-1)^2 = \underline{\underline{1}}$

11) (a) $\lim_{x\to 1}\frac{\ln x}{x-1} = \lim_{x\to 1}\frac{\frac{1}{x}}{1} = \underline{\underline{1}}$

(b) $\lim_{x\to\infty}\frac{\ln x}{x} = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \underline{\underline{0}}$

(c) $\lim_{x\to 0^+}\frac{\ln x}{\sqrt{x}} = \lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{1}{2}x^{-\frac{1}{2}}} = \lim_{x\to 0^+}\frac{\sqrt{x}}{2x}$

$= \lim_{x\to 0^+}\frac{\frac{1}{2}x^{-\frac{1}{2}}}{2(1)} = \lim_{x\to 0^+}\frac{1}{4\sqrt{x}} = \underline{\underline{\infty}}$

(d) $\lim_{x\to\infty} \frac{(\ln x)^3}{x^2}$

$= \lim_{x\to\infty} \frac{3(\ln x)^2 \frac{1}{x}}{2x}$

$= \lim_{x\to\infty} \frac{6\ln x(\frac{1}{x^2}) - 3(\ln x)^2 \frac{1}{x^2}}{2}$

$= \lim_{x\to\infty} \left(\frac{3\ln x}{x^2} - \frac{3}{2x^2}(\ln x)^2\right)$

$\lim_{x\to\infty} \frac{3\ln x}{x^2}$ $\quad$ $\lim_{x\to\infty}\left(-\frac{3}{2x^2}(\ln x)^2\right)$

$= \lim_{x\to\infty} \frac{3(\frac{1}{x})}{2x}$ $\quad$ $= \lim_{x\to\infty}\left(-\frac{6\ln x(\frac{1}{x})}{4x}\right)$

$= \lim_{x\to\infty} \frac{3}{2x^2}$ $\quad$ $= \lim_{x\to\infty} -\frac{3}{2}\frac{\ln x}{x^2}$

$= 0$ $\quad$ $= 0$

$\lim_{x\to\infty} \frac{(\ln x)^3}{x^2} = \underline{\underline{0}}.$

(e) $\lim_{x\to 2^-} \frac{\ln x}{\sqrt{2-x}} = \underline{\underline{\infty}}$

(f) $\lim_{x\to\infty} \frac{\ln(\ln x)}{\sqrt{x}}$

$= \lim_{x\to\infty} \frac{\frac{1}{\ln x}\frac{1}{x}}{\frac{1}{2}x^{-\frac{1}{2}}}$

$= \lim_{x\to\infty} \frac{2\sqrt{x}}{x\ln x}$

$= \lim_{x\to\infty} \frac{2}{\sqrt{x}\ln x} = \underline{\underline{0}}$

(g) $\lim_{x\to\infty} \frac{\ln(1+e^x)}{5x}$

$= \lim_{x\to\infty} \frac{\frac{1}{1+e^x}e^x}{5}$

$= \lim_{x\to\infty} \frac{1}{5}\frac{e^x}{1+e^x}$

$= \lim_{x\to\infty} \frac{e^x}{5e^x}$

$= \underline{\underline{\frac{1}{5}}}$

(h) $\lim_{x\to 0^+} \sqrt{x}\ln x$

$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sqrt{x}}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-\frac{3}{2}}}$

$= \lim_{x\to 0^+} -\frac{2x^{\frac{3}{2}}}{x}$

$= \lim_{x\to 0^+} -2x^{\frac{1}{2}} = \underline{\underline{0}}$

(c) $\lim_{x\to\infty} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}}$

$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}}$

$= \underline{\underline{1}}$

12) (a) $h(x) = (x^2-1)^3$

$h'(x) = 3(x^2-1)^2(2x)$

$= 6x(x^2-1)^2$

$h'(x) = 0$ for $x = 0, \pm 1$

$h''(x) = 6(x^2-1)^2 + 12x(x^2-1)2x$

$= 6(x^2-1)^2 + 24x^2(x^2-1)$

$= 6(x^2-1)(x^2-1+4x^2)$

$= 6(x^2-1)(5x^2-1)$

$h''(x) = 0$ for $x = \pm 1, \pm\frac{1}{\sqrt{5}}$

x-value	$h'(x)$	$h''(x)$	$h(x)$
$x<-1$	−	+	decr., concave ↑
-1			POI
$-1<x<-\frac{1}{\sqrt{5}}$	−	−	decr., concave ↓
$-\frac{1}{\sqrt{5}}$			POI
$-\frac{1}{\sqrt{5}}<x<0$	−	+	decr., concave ↑
0			local min
$0<x<\frac{1}{\sqrt{5}}$	+	+	incr., concave ↑
$\frac{1}{\sqrt{5}}$			POI
$\frac{1}{\sqrt{5}}<x<1$	+	−	incr., concave ↓
1			POI
$x>1$	+	+	incr., concave ↑

$h(0) = -1$

(b) $P(x) = x\sqrt{x^2+1}$

$$P'(x) = \sqrt{x^2+1} + x\left(\tfrac{1}{2}\right)(x^2+1)^{-\frac{1}{2}}2x$$
$$= (x^2+1)^{-\frac{1}{2}}\left[(x^2+1)+x^2\right]$$
$$= \frac{2x^2+1}{\sqrt{x^2+1}}$$

~~P'(x) = 0 or is undefined for~~

~~x =~~ $P'(x)$ is never 0

$$P''(x) = \frac{4x\sqrt{x^2+1} - \frac{1}{2}(x^2+1)^{-\frac{1}{2}}2x(2x^2+1)}{x^2+1}$$
$$= \frac{1}{\sqrt{x^2+1}}\,\frac{\left(4x(x^2+1) - x(2x^2+1)\right)}{x^2+1}$$
$$= x(x^2+1)^{-\frac{3}{2}}\left(4(x^2+1)-(2x^2+1)\right)$$
$$= x(x^2+1)^{-\frac{3}{2}}(2x^2+2)$$
$$= 2x(x^2+1)^{-\frac{1}{2}}$$
$$= \frac{2x}{\sqrt{x^2+1}}$$

$P''(x) = 0$ for $x = 0$

x-value	$P'(x)$	$P''(x)$	$P(x)$
$x<0$	+	−	incr., concave ↓
~~x>0~~ 0			POI
$x>0$	+	+	incr., concave ↑

(c) $P(x) = x\sqrt{x+1}$

$$P'(x) = \sqrt{x+1} + \tfrac{1}{2}x(x+1)^{-\frac{1}{2}}$$
$$= (x+1)^{-\frac{1}{2}}\left[(x+1)+\tfrac{1}{2}x\right]$$
$$= (x+1)^{-\frac{1}{2}}\left(\tfrac{3}{2}x+1\right)$$
$$= \frac{\frac{3}{2}x+1}{\sqrt{x+1}}$$

$P'(x) = 0$ or is undefined for

$x = -\frac{2}{3}, -1$

$$P''(x) = -\tfrac{1}{2}(x+1)^{-\frac{3}{2}}\left(\tfrac{3}{2}x+1\right) + (x+1)^{-\frac{1}{2}}\tfrac{3}{2}$$
$$= (x+1)^{-\frac{3}{2}}\left(-\tfrac{3}{4}x - \tfrac{1}{2} + \tfrac{3}{2}(x+1)\right)$$
$$= (x+1)^{-\frac{3}{2}}\left(\tfrac{3}{4}x+1\right)$$
$$= \frac{\frac{3}{4}x+1}{(x+1)^{\frac{3}{2}}}$$

$P''(x) = 0$ for $x = -\frac{4}{3}$

x-value	$P'(x)$	$P''(x)$	$P(x)$
$-1<x<-\frac{2}{3}$	−	−	decr., concave ↑
$-\frac{2}{3}$		POI	local min
$x>-\frac{2}{3}$	+	+	incr., concave ↑

$P\left(-\frac{2}{3}\right) = -\frac{2}{3}\sqrt{\frac{1}{3}} \approx -0.385$

(d) $Q(x) = x^{\frac{1}{3}}(x+3)^{\frac{2}{3}}$

$$Q'(x) = \tfrac{1}{3}x^{-\frac{2}{3}}(x+3)^{\frac{2}{3}} + \tfrac{2}{3}x^{\frac{1}{3}}(x+3)^{-\frac{1}{3}}$$
$$= \tfrac{1}{3}x^{-\frac{2}{3}}(x+3)^{-\frac{1}{3}}\left[(x+3)+2x\right]$$
$$= \tfrac{1}{3}x^{-\frac{2}{3}}(x+3)^{-\frac{1}{3}}(3x+3)$$
$$= (x+1)x^{-\frac{2}{3}}(x+3)^{-\frac{1}{3}}$$

$Q'(x) = 0$ or is undefined for

$x = -1, 0, -3$

$$Q''(x) = x^{-\frac{2}{3}}(x+3)^{-\frac{1}{3}} - \tfrac{2}{3}(x+1)x^{-\frac{5}{3}}\cdot(x+3)^{-\frac{1}{3}} - \tfrac{1}{3}(x+1)x^{-\frac{2}{3}}(x+3)^{-\frac{4}{3}}$$
$$= \tfrac{1}{3}x^{-\frac{5}{3}}(x+3)^{-\frac{4}{3}}\left[3x(x+3) - 2(x+1)(x+3) - (x+1)x\right]$$
$$= \tfrac{1}{3}x^{-\frac{5}{3}}(x+3)^{-\frac{4}{3}}(-6)$$
$$= -2x^{-\frac{5}{3}}(x+3)^{-\frac{4}{3}}$$

$Q''(x)$ is never 0

x-value	$Q'(x)$	$Q''(x)$	$Q(x)$
$x<-3$	+	+	incr., concave up
-3			local max.
$-3<x<-1$	−	+	decr., concave up
-1		+	local min.
$-1<x<0$	+	+	incr., concave up
0			POI
$x>0$	+	−	incr., concave up

$Q(-3) = \sqrt[3]{-3}\,(0) = 0$

$Q(-1) = -1(-2)^{\frac{2}{3}}$

$= -\sqrt[3]{4} \approx -1.587$