1. Each of the following functions gives the equation of motion for a particle, where s is in meters and t is in seconds. Find the velocity and acceleration as functions of t.

   (a) $s(t) = t^3 - 3t$

   (b) $s(t) = t^2 - t + 1$

   (c) $s(t) = At^2 + Bt + C$

   (d) $s(t) = 2t^3 - 7t^2 + 4t + 1$

2. For each of the following, find the equation of the tangent line to the given curve at the given point.

   (a) $y = x + \frac{4}{x}$, $(2, 4)$

   (b) $y = x^{\frac{5}{2}}$, $(4, 32)$

   (c) $y = x + \sqrt{x}$, $(1, 2)$

3. The *normal line* to a curve C at a point P is the line that passes through P and is perpendicular to the tangent line to C at P. For each of the following, find the equation of the normal line to the curve at the given point.

   (a) $y = 1 - x^2$, $(2, -3)$

   (b) $y = \sqrt[3]{x}$, $(-8, -2)$

   (c) $y = f(x)$, $(a, f(a))$

4. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

5. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

6. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

7. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

8. At what point on the curve $y = x^4$ does the normal line have slope 16?

9. Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time?

10. Let

$$f(x) = \begin{cases} 2 - x & \text{if } x \leq 1, \\ x^2 - 2x + 2 & \text{if } x > 1. \end{cases}$$

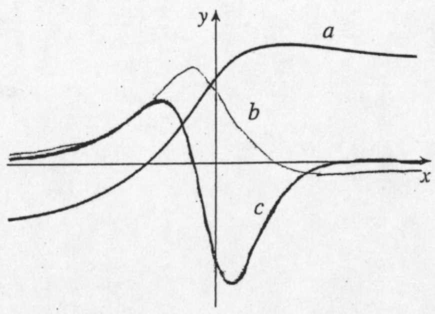
Is f differentiable at 1? Sketch the graphs of f and f'.

11. Let

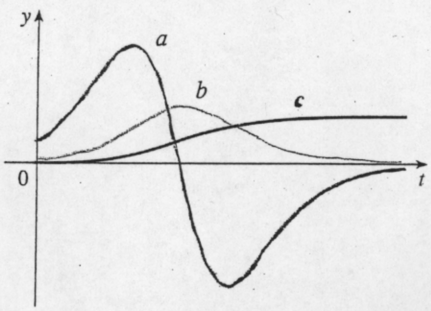
$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2, \\ mx + b & \text{if } x > 2. \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

12. The following figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



13. The following figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



1) (a) $s(t) = t^3 - 3t$

$s'(t) = 3t^2 - 3$

$s''(t) = 6t$

(b) $s(t) = t^2 - t + 1$

$s'(t) = 2t - 1$

$s''(t) = 2$

(c) $s(t) = At^2 + Bt + C$

$s'(t) = 2At + B$

$s''(t) = 2A$

(d) $s(t) = 2t^3 - 7t^2 + 4t + 1$

$s'(t) = \cancel{8}6t^2 - 14t + 4$

$s''(t) = 12t - 14$

2) (a) $y = x + \frac{4}{x}$, $(2,4)$

$y' = 1 - \frac{4}{x^2}$

$y'(2) = 1 - \frac{4}{4} = 0$

$y = 4$

(b) $y = x^{\frac{5}{2}}$, $(4, 32)$

$y' = \frac{5}{2} x^{\frac{3}{2}}$

$y'(4) = \frac{5}{2}(4)^{\frac{3}{2}}$

$= \frac{5}{2}(8) = 20$

$y = 20x + b$

$32 = 20(4) + b$

$32 = 80 + b$

$b = -48$

$y = 20x - 48$

(c) $y = x + \sqrt{x}$, $(1,2)$

$y' = 1 + \frac{1}{2}x^{-\frac{1}{2}}$

$y'(1) = 1 + \frac{1}{2} = \frac{3}{2}$

$y = \frac{3}{2}x + b$

$2 = \frac{3}{2} + b \Rightarrow b = \frac{1}{2}$

$y = \frac{3}{2}x + \frac{1}{2}$

3)(a) $y = 1 - x^2$, $(2, -3)$

$y' = -2x$

$y'(2) = -4$

$y = \frac{1}{4}x + b$

$-3 = \frac{1}{4}(2) + b = \frac{1}{2} + b$

$b = -\frac{7}{2}$

$y = \frac{1}{4}x - \frac{7}{2}$

(b) $y = \sqrt[3]{x}$, $(-8, -2)$

$y' = \frac{1}{3}x^{-\frac{2}{3}}$

$y'(-8) = \frac{1}{3}(-8)^{-\frac{2}{3}} = \frac{1}{3}(4)^{-1} = \frac{1}{12}$

~~$-2 = \frac{1}{12}(-8) + b \Rightarrow b = -\frac{4}{3}$~~

~~$y = \frac{1}{12}x - \frac{4}{3}$~~

$y = -12x + b$

$-2 = -12(-8) + b$

$-2 = 96 + b$

$-98 = b$

$y = -12x - 98$

(c) $y = f(x)$, $(a, f(a))$

$y' = f'(x)$

$y'(a) = f'(a)$

$y = -\frac{1}{f'(a)}x + b$

$f(a) = -\frac{1}{f'(a)}(a) + b$

$b = f(a) + \frac{a}{f'(a)}$

$y = -\frac{1}{f'(a)}x + f(a) + \frac{a}{f'(a)}$

4) $3x - y + 6 = 0$

$y = 3x + 6$ slope 3

$y = x\sqrt{x} = x^{\frac{3}{2}}$

$y' = \frac{3}{2}x^{\frac{1}{2}}$

$y' = 3$ when $\frac{3}{2}x^{\frac{1}{2}} = 3$

or $x^{\frac{1}{2}} = 2$

or $x = 4$

$(4, 8), (4, -8)$

5) $f(x) = 2x^3 - 3x^2 - 6x + 87$

$f'(x) = \cancel{8}6x^2 - 6x - 6$

$= 6(x^2 - x - 1)$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

6) $y = x^3 - x^2 - x + 1$

$y' = 3x^2 - 2x - 1$

$= (3x+1)(x-1)$

$x = -\frac{1}{3}, 1$

$(-\frac{1}{3}, \frac{32}{27}), (1, 0)$

7) $y = 6x^3 + 5x - 3$

$y' = 18x^2 + 5$

$y' = 4 \Rightarrow 18x^2 + 5 = 4$

$\Rightarrow 18x^2 = -1$

impossible since

$x^2 \geq 0$

8) $y = x^4$

$y' = 4x^3$

$y' = -\frac{1}{16}$

$\Rightarrow 4x^3 = -\frac{1}{16}$

$\Rightarrow x^3 = -\frac{1}{64}$

$\Rightarrow x = -\frac{1}{4}$

$(-\frac{1}{4}, \frac{1}{256})$

9) $y = x - x^2$

$y' = 1 - 2x$

$y'(1) = 1 - 2 = -1$

normal line: $y = x + b$

$0 = 1 + b$

$b = -1$

$y = x - 1$

$x - 1 = x - x^2$

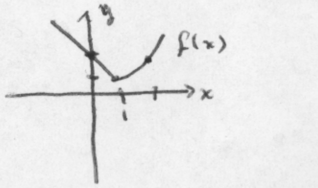
$-1 = -x^2$

$1 = x^2$

$x = \pm 1$

$(1, 0), (-1, -2)$

10) $f(x) = \begin{cases} 2 - x & \text{if } x \leq 1, \\ x^2 - 2x + 2 & \text{if } x > 1 \end{cases}$



$x^2 - 2x + 2$

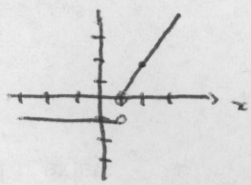
$= (x^2 - 2x + 1) + 1$

$= (x-1)^2 + 1$

f is not differentiable at 1

for $x < 1$, $f'(x) = -1$

for $x > 1$, $f'(x) = 2x - 2$

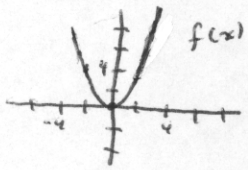


11)

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx+b & \text{if } x > 2 \end{cases}$$

$f(x)$ is differentiable for $x<2$ and for $x>2$

for $m=4$, b $=-4$ ~~any real number~~, f is differentiable everywhere



12) c derivative of b
b derivative of a
c second derivative of a

f a
f' b
f'' c

13) a derivative of b
b derivative of c

position c
velocity b
acceleration a