Math 1, Fall 2003
Goals for Week 5: October 20-24, 2003

Raising Functions to Positive Powers: You should know that, given a function $f(x)$, we can get a new function $g(x)$ by raising $f(x)$ to the nth power. Given $g(x) = (f(x))^n$, you should be able to apply the power rule to differentiate $g(x)$. You should be able to recognize when one function $g(x)$ is the power of another function $f(x)$, and be able to apply the power rule to $g(x)$. You should understand the power rule of differentiation and the formula we use to differentiate positive power functions are consistent with each other.

Products of Functions: Given two functions $f(x)$ and $g(x)$, you should know that we can build a new function $h(x) = f(x)g(x)$ by multiplying $f(x)$ and $g(x)$ together. You should be able to apply the product rule to differentiate $h(x)$. You should be able to recognize when one function $k(x)$ is the product of two other functions, and you should be able to differentiate $k(x)$ using the product rule. You should be familiar with some examples of why the derivative of the product of two functions is not equal the product of the derivatives of those two functions.

Vertical Stretches of Functions: You should know what a vertical stretch of a function is, both graphically and algebraically. You should be able to describe the effect you changing the parameter a from a large positive number to a large negative number. In particular, you should know what the graph of the vertical stretch of a function looks like when $a > 1$, when $a = 1$, when $0 < a < 1$, when $a = 0$, when $-1 < a < 0$, when $a = -1$, and when $a < -1$. You should know how to differentiate $g(x)$ if $g(x)$ is the vertical stretch of a function $f(x)$.

Horizontal Stretches of Functions: You should know what a horizontal stretch of a function is, both graphically and algebraically. You should be able to describe the effect you changing the parameter a from a large positive number to a large negative number. In particular, you should know what the graph of the horizontal stretch of a function looks like when $a > 1$, when $a = 1$, when $0 < a < 1$, when $a = 0$, when $-1 < a < 0$, when $a = -1$, and when $a < -1$. You should know how to differentiate $g(x)$ if $g(x)$ is the horizontal stretch of a function $f(x)$.

Vertical Shifts of Functions: You should know what a vertical shift of a function is, both graphically and algebraically. You should be able to describe the effect you changing the parameter b from a positive number to a negative number. In particular, you should know what the graph of the vertical shift of a function looks like when $b > 0$, when $b = 0$, and when $b < 0$. You should know how to differentiate $g(x)$ if $g(x)$ is the vertical shift of a function $f(x)$.

Horizontal Shifts of Functions: You should know what a horizontal shift of a function is, both graphically and algebraically. You should be able to describe the effect you changing the parameter b from a positive number to a negative number. In particular, you should know what the graph of the horizontal shift of a function looks like when $b > 0$, when $b = 0$, and when $b < 0$. You should know how to differentiate $g(x)$ if $g(x)$ is the horizontal shift of a function $f(x)$.

The Composition of Two Functions You should know what the composition of two functions is, and you should have an image in your mind of how two functions can be applied one after another to get a single function. You should be familiar with the notation $g \circ f$ for the composition of two functions, and you should know that $g \circ f$ means that you apply $f(x)$ first, then $g(x)$, and not vice versa. Given the formulae of two real valued functions $f(x)$ and $g(x)$ whose domains are the set of real numbers, you should be able to find a formula for the composition $(g \circ f)(x)$. You should know examples of functions $f(x)$ and $g(x)$ for which $(g \circ f)(x)$ does not equal $(f \circ g)(x)$.

Stretches and Shifts as Compositions You should be able to write any vertical stretch, horizontal stretch, vertical shift, or horizontal shift of a function $f(x)$ as a composition of $f(x)$ with some other function. You should have some understanding of the relationship between the order of the above compositions and the direction (horizontal or vertical) of a stretch or shift.

Seeing Functions as Compositions Given a sufficiently-complicated function $h(x)$, you should be able to determine the formulae for two functions $f(x)$ and $g(x)$ such that $h(x) = (g \circ f)(x)$. You should become familiar with the terminology "inner function" and "outer function." You should be familiar with the idea that there may be more than one way to break up a function into the two other functions, and you should be able to recognize functions which are the composition of three or more other functions.

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Lecture Notes for Week 5: Lectures 11, Lecture 12, and Lecture 13

Homework for Week 5: Homework 9 and Homework 10