**Math 1, Fall 2003**
**Goals for Week 4: October 13-17, 2003**

**The Meaning of the First Derivative:** You should that, if the first derivative of a function at a point is positive, then that function is increasing at that point, and if the first derivative of a function at a point is negative, then that function is decreasing at that point. You should know that, if a function has a local maximum or a local minimum at a point, then the derivative of the function at that point must be zero. You should know what a critical point is, and you should know be able to give an example of when a function can have a critical point without having a local maximum or at a local minimum.

**The Meaning of the Second Derivative:** You should know what it means for a function to be concave up or concave down at a point. You should know that when the second derivative of a function at a point is positive, then that function is concave up at that point, and that when the second derivative of a function at a point is negative, then that function is concave down at that point. You should know that, when the second derivative of a function at a point is zero, then we do not learn anything new about the shape of the graph of that function at that point.

**Critical Points and the Second Derivative Test:** You should know what the Second Derivative Test is and how to use it. You should know that if the second derivative of a function at a critical point is positive, then that critical point is a local minimum, and that if the second derivative of a function at a critical point is negative, then the critical point is a local maximum. You should know that if the second derivative of a function at a critical point is zero, then we learn no new information about that critical point. You should have an intuitive understanding of why the Second Derivative Test works.

**Inflection Points:** You should know what an inflection point is. You should know that, if a point on the graph of a function is an inflection point, then the second derivative of the function at that point must be zero. You should know an example of a function which has a second derivative of zero at a point, and yet that point is not an inflection point.

**Radians:** You should know how to define the number of radians in an angle. You should know that the number of radians in an angle is independent of the radius of the circle used to draw that angle. You should know how to convert from degrees to radians and back to degrees. You should know the conversion to radians for the common angles (see notes). You should know what it means to have an angle of greater than $2\pi$ radians, and what it means to have an angle of negative radians, and how to find the endpoint of the arc defined by that angle (starting at the positive $x$-axis) for the common angles.

**Sine and Cosine:** You should know how to find the sine and cosine of an acute angle. You should know the Pythagorean Theorem. You should be able to define the sine and cosine of any number of radians. You should know how the endpoint of the arc defined by an angle (starting at the positive $x$-axis) is related to the sine and cosine of that angle. You should have memorized the sines and cosines of the common angles (see notes), and you should know how to use the unit circle to remind you of the sines and cosines of those common angles. You should know the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$.

**The Graphs of Sine and Cosine** You should be able to accurately sketch the graphs of the functions $\sin x$ and $\cos x$. You should be able to graphically find the local maxima, local minima, and inflection points of the graphs of these two functions.

**The Properties of Sine and Cosine** You should know the main algebraic properties of $\sin x$ and $\cos x$ and how to algebraically represent those properties. Specifically, you should know that

- the functions $\sin x$ and $\cos x$ are periodic, what it means to be periodic, how to represent periodicity with a formula, and what the periods of $\sin x$ and $\cos x$ are.
- the functions $\sin x$ and $\cos x$ are bounded, what it means to be a bounded function, what the ranges of $\sin x$ and $\cos x$ are, and what the amplitudes of $\sin x$ and $\cos x$ are.

- the functions $\sin x$ and $\cos x$ are shifts of each other, what it means for one function to be a shift of another, and how to represent the fact that $\sin x$ and $\cos x$ are shifts of each other algebraically.
- the function $\sin x$ is a odd function, the function $\cos x$ is an even function, and how to represent these two facts with formulae.

**The Derivatives of Sine and Cosine** You should know what the derivatives of $\sin x$ and $\cos x$ are and how the functions are related by derivatives. You should be able to find the higher derivatives of $\sin x$ and $\cos x$, and you should know that there is a repeating pattern to the higher derivatives of $\sin x$ and $\cos x$.

**Generalized Sine and Cosine Functions** You should know the general forms of the generalized sine and cosine functions. You should know how to find the amplitude and the period of generalized sine and cosine functions. You should be aware of the effect of increasing and decreasing $A$ and $k$ on the graphs of generalized sine and cosine functions. You should be able to calculate the derivatives of generalized sine and cosine functions.

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**Lecture Notes for Week 4:** Lectures 8, Lecture 9, and Lecture 10

**Homework for Week 4:** Homework 7 and Homework 8