

Math 17  
Winter 2015  
Final Project  
Preliminary Deadlines

Your final project is to write a paper, and give a short presentation to the class, about some topic connected with Hilbert's tenth problem.

Your topic may have a mathematical, philosophical, or historical focus, although your paper may not be entirely mathematics-free. That is, your topic should require you to explain some mathematics, although it doesn't have to be mathematics that you produce yourself.

You may work alone or in a group of 2 or 3 people. The scope of your project should be appropriate to the size of your group. For your class presentation, each person should present for about 10 minutes. For your paper, each person should write several pages. (Exactly how many depends partly upon the nature of your topic.) Each person will be graded on their own writing and presentation; the group as a whole will be graded on the overall quality of the project.

Examples of suitable topics:

Choose another of Hilbert's problems. Explain what the problem is about, why it was an important problem in 1900, what the current status of the problem is (with a discussion of relevant mathematical results), and what related questions still remain open.

Choose a mathematician connected with Hilbert's tenth problem, and write about their life and mathematical work. It's best to choose someone who hasn't already been written on extensively. For example, the first mention in print of the possibly insolubility of Hilbert's tenth problem (that I can find) is in a review of a talk by Rosza Péter, an early researcher in recursion theory (the theory of Turing machine computability). For another example, Emil Post, another such researcher, strongly believed in this possibility, and communicated that belief to his student, Martin Davis.

Church's thesis (or Turing's thesis, or the Church-Turing thesis, or the Church-Turing-Markov thesis), the claim that a decision problem that is not Turing decidable is therefore intuitively undecidable, is central to the claim that Hilbert's tenth problem has been proven to have no solution. Church's thesis is essentially a philosophical claim. Compare the justification of Church's thesis given in Turing's original paper with that given in our textbook, or in Enderton's textbook *A Mathematical Introduction to Logic*. Which seems stronger? What (if anything) do contemporary philosophers have to say on the subject?

Choose an issue addressed in the exercises of the textbook, for example, giving Diophantine representations with the smallest possible number of variables. Do some of the relevant exercises. Find out what relevant mathematical research has been done. (For example, we know that the version of Hilbert's tenth problem with only one variable is solvable. How many variables are needed before you get unsolvability? What is the history of work on this problem?)

The textbook proves that if a set  $X$  and its complement are both Turing semidecidable, then  $X$  is decidable, using, in the proof, the equivalence of semi decidability and Diophan-

teness. The author comments that it is possible to prove this directly, by showing how, given semidecision machines for  $X$  and its complement, to produce a decision machine for  $X$ . Work out the details of this argument.

**Due Monday, February 9:** Tell me your choice of topic and, if you wish to work with others, the members of your group. You should meet with me beforehand to discuss your choice.

**Due Monday, February 16:** Submit a project proposal. Your final paper does not have to be exactly as the proposal describes, although it's probably a good idea to consult with me if you are making major changes. Nevertheless, you should have a reasonably detailed plan to work with.

Ingredients of a project proposal:

What is the subject of your paper? What questions will you address? What do you expect your paper to contain?

What mathematics do you expect to present? Will you work out any of the results yourself, or will all the mathematics be results you found elsewhere and are presenting?

What sources do you plan to use? For each source, give bibliographic information, and say something about what you expect to get from this source, and why it may be useful.

Who will write what part of the paper? Each person should present some significant mathematics. You may structure your paper in any way that works, from two essentially separate chapters to a paper of multiple sections with alternating authors, as long as you make it clear to me who wrote which parts.

**Optional: Due Monday, February 23:** Submit a preliminary draft for comments.

**Due Monday, March 2:** First drafts.

Oral presentations. You should be prepared to present any day this week. For your oral presentation to the class, each person should speak for about 10 minutes. Remember that people may have questions, which will take up some of your time.

**Due Saturday, March 14, 4:00 PM:** Final version of paper. This is the date reserved for our final exam; since we aren't having an exam, the final paper is due today.