Math 17
Winter 2015
Project 1

Assignment:

For your first project, you will write a paper showing that the following restricted version of Hilbert's Tenth Problem has a positive solution.

> Devise a process according to which it can be determined by a finite number of operations whether a given Diophantine equation with one unknown and integer coefficients has integer solutions.

You will do this by describing an algorithm, along the lines suggested by problems (2) and (3) on the first written homework assignment, that, given a polynomial

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

with integer coefficients, answers the question of whether or not $P(x)$ has any integer roots.

Your paper must describe the algorithm and prove that it works. Your description of the algorithm must be precise and complete, but it need not look like a computer program or a flowchart. For example, the following is the description of an algorithm to determine whether an integer n is a perfect square (from the class notes from January 5):

> Given an integer n, if $n < 0$ then answer NO, n is not a perfect square, and if $n = 0$, answer YES, n is a perfect square. If $n > 0$, compute the squares of the natural numbers, 0^2, 1^2, 2^2, 3^2, ..., until you reach a natural number k such that either $k^2 = n$ or $(k-1)^2 < n < k^2$. If $k^2 = n$, answer YES, n is a perfect square, and if $(k-1)^2 < n < k^2$, answer NO, n is not a perfect square.

Of course, this is only a description of the algorithm, not a proof that it works.

Any calculus facts that you use should be justified, via some combination of proof and reference to standard calculus theorems and techniques. For those standard theorems and techniques, you must give a citation, and include a reference in a list of references at the end of your paper. You must also acknowledge any other sources you consult, any help you receive, and any person you work with.

Your paper should be written in the style of a mathematics paper or article. The sample paper "Limits of Polynomials at Infinity," which you can download from the course webpage, is an example of that style. You could also go to the math collection in Berry Library, and look at a current issue of *Mathematics Magazine*.

You might well wonder what you are allowed to assume.

Your intended reader is a student who does not understand this material as well as you do. Loosely, you may assume your reader knows arithmetic, high school algebra, and precalculus. Things you use from calculus should be explained and a citation given.

In the world of professional mathematics, many of the calculus facts you are likely to use, such as the formula for the derivative of a polynomial, are considered common knowledge and would not necessarily come with a citation or explanation. However, you aren't writing for professional mathematicians.

As far as your algorithm is concerned, you don't have to go into details about arithmetic and straightforward symbolic manipulation. That is, you can include steps like "evaluate $P(x)$ for $x = a$" (assuming that you have already explained what a is), or "find the derivative of P" (assuming you have already given the formula for the derivative of a polynomial) without explaining how to do those things.

Important Note:

On your preliminary assignment, I did not in general make comments about the entries on your reference list. That does not mean you had the details correct.

At the URL

http://bcs.bedfordstmartins.com/resdoc5e/RES5e_ch11_s1-0003.html

you can find some general information about citing sources in scientific writing.

General practice in mathematical writing calls for beginning each entry with the author's name (or the name of the group or institution that stands in as author, if the authorship is anonymous), and organizing the reference list alphabetically by author's last name. I don't insist on this format, but I do insist that you include an author for every source, including internet sources.

In fact, for an online source, you should include all the information you include for a printed source, insofar as possible: Author, title, publication information, publication or copyright date. There are two other things that you should include for an online source; the URL, and the date at which you retrieved, or cited, the information.

Remember that citation serves several purposes. Some reasons to cite are to avoid presenting others' work as your own, to give credit where credit is due, to enhance the authority and credibility of your writing, and to allow your reader to verify what you have said or to learn more about it.

Grading: Your project will be graded based on the following criteria:

1. Your algorithm is correct.
2. The description of your algorithm is precise and complete.
3. The description of your algorithm is clear.
4. The proof that your algorithm works is correct.
5. The proof that your algorithm works is precise and complete.
6. The proof that your algorithm works is clear.
7. Your paper as a whole is organized so as to present your argument in a logical and coherent fashion, and written in such a way as to make the logical structure of the paper clear to the reader.
8. Your use of language, including but not limited to technical mathematical language, is precise and correct.
9. Your prose is easy to read.
10. Your style is clear. (Bonus: Your style is graceful, or entertaining, or has other pleasing qualities. This is a bonus; clarity is paramount.)
11. Your paper includes an introduction or discussion that gives some context, motivation, or other reason a reader should be interested. (This does not have to be long.)
12. Everything that should be cited or acknowledged is appropriately cited or acknowledged.
13. Your paper is in an appropriate format, with title, abstract, and list of references.
14. Your reference list is complete, and includes all necessary details.
15. Your preliminary assignments and first draft were complete and were turned in on time.

Timeline of Assignments: You have already submitted your first preliminary assignment. A copy of that assignment follows on the next pages.

Preliminary Assignment 2: Type up preliminary assignment 1, making corrections and improvements in response to my comments on your paper and to the "important note" above, in LaTeX. This is due by the beginning of class on Tuesday, January 20.

First Draft: A first draft of your paper (typed!) is due by the beginning of class on Monday, January 26. This should be a complete first draft, the best paper you can produce at the time. (Your final draft will, one hopes, be an improvement based on my response to your first draft.)

Final Draft: The final draft of your paper is due by the beginning of class on Monday, February 2. This is the paper that will be graded.

Preliminary Assignment 1

This preliminary assignment is graded credit or no credit (or partial credit for late or incomplete submissions). To get full credit, you need only submit a complete assignment, with all help and sources correctly acknowledge and cited, by the due date. This grade will be part of your grade for your first project. When I return your assignment, pay attention to comments and corrections, because your final draft will, of course, be graded.

This assignment is due at the beginning of class on Monday, January 12.

Assignment: Write up a proof of the following proposition as if it were part of a paper. You need not include an abstract, introduction, context, discussion, etcetera; simply give the statement of the proposition and give a proof. You must, however, correctly acknowledge and/or cite any help you receive and any sources you use, and include a list of references. (You should also include a title of some kind, and, of course, your name.)

In particular, if you use any theorems or facts from calculus, you must find them in some appropriate source (such as your calculus textbook), cite that source, and include that source in your list of references.

The sample paper *Lmits of Polynomials at Infinity* is an example of how a mathematical paper should be written.

This assignment may be written by hand. Your second preliminary assignment will be to type this in LaTeX, making changes in response to any corrections or comments I make, and will be due on Tuesday, January 20. We will have an introduction to LaTeX in class on Friday, January 16.

Proposition: Suppose $P(x)$ is a polynomial with real number coefficients, and a, b, and c are real numbers such that $a < b < c$, $P(a) > 0$, $P(b) \leq 0$, and $P(c) > 0$. Then there is a number x in the interval $[a, c]$ such that $P'(x) = 0$.