Math 17
Winter 2015
Written Exercises Assigned January 5

These exercises are due at the beginning of class on Friday, January 9.

Honor principle statement: You may always work together on written exercises. You may get help from anyone. You may consult any source.

You must write up your answers yourself, and you must acknowledge anyone you worked with and any source you consulted. An informal note at the beginning or end of the problem set is sufficient; for example, "I worked on these exercises together with John Lee and Mary Lincoln, we got help with exercise 3 from Chris Estes, and I looked up the statement of the Mean Value Theorem on Wikipedia."

In formal work (for this course, this means your two projects), formal citations (and more academically credible sources than Wikipedia) are required.

This honor principle statement applies to all the weekly homework exercises.

1. Give an example, other than those from class or from the text, of a problem (either a decision problem or a computation problem) that can be reduced to another problem. Explain your example. (That is, explain what each problem asks for, and why it's a decision or computation problem. You need not give a solution; that is, you need not solve your decision or computation problems. You do need to explain how to reduce one to the other, and why that reduction works.)

   Simple is okay here. For example, the problem of multiplying fractions can be reduced to the problem of multiplying integers, because you can multiply two fractions by multiplying their numerators and multiplying their denominators.

   However, the problems must be computation or decision problems as we defined them. For example, the problem of multiplying real numbers does not fit our definition, because you cannot specify an arbitrary real number by giving finitely much information.

2. Show that this problem:

   > Given a polynomial $P(x)$ with integer coefficients and one unknown x, determine whether $P(x)$ has any integer roots;

   can be reduced to this problem:

   > Given a polynomial $P(x)$ with integer coefficients and one unknown x, find an integer $b \geq 0$ such that all real roots of $P(x)$ (if any) are found in the interval $[-b, b]$.

   (Here a "real root" means a root that is a real number rather than a complex number.)

3. Show that this problem:

   > Given a polynomial $P(x)$, with integer coefficients and one unknown x, of degree 348, find an integer $b \geq 0$ such that all real roots of $P(x)$ (if any) are found in the interval $[-b, b]$.

   can be reduced to this problem:

   > Given a polynomial $P(x)$, with integer coefficients and one unknown x, of degree 347, find an integer $b \geq 0$ such that all real roots of $P(x)$ (if any) are found in the interval $[-b, b]$.

   Hint: Think about what the graph of a polynomial looks like, and what you learned in calculus about curve sketching.

   In this exercise, you may use any facts you know from calculus, without justifying them. However, if you need to consult a source or a person to remind yourself of those facts, you must of course acknowledge that source or person.

4. In section 1.3 of the textbook, the following two problems are considered:

   (a) Hilbert's Tenth Problem, original (integer) version: Given a Diophantine equation, determine whether there are integer solutions.

   (b) Hilbert's Tenth Problem, natural number version: Given a Diophantine equation, determine whether there are natural number solutions.

   We wish to show that the integer version (a) is unsolvable, by showing that the natural number version (b) is unsolvable. In order to do this, do we need to show that (a) is reducible to (b), or that (b) is reducible to (a)? Explain.

Note: The answers to exercises 2 and 3 will be used in your first project.

Second note: You should try reading Section 1.3, and make sure you understand how each problem is reduced to the other.