Midterm 2 : In-class part

1\. (a.) $\mathcal{B} = \{1, 7, 5\}$

(b.) $\mathcal{N} = \{6, 2, 3, 4, 8\}$

(c.) $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & -1 \end{bmatrix}$

(d.) $N = \begin{bmatrix} 1 & -1 & 2 & 3 & 0 \\ 0 & 2 & 4 & 4 & 0 \\ 0 & -8 & 5 & 7 & 1 \end{bmatrix}$

(e.) $B^{-1}N = \begin{bmatrix} 1 & -1 & 2 & 3 & 0 \\ -2 & 4 & 0 & -2 & 0 \\ 3 & 5 & 1 & 2 & -1 \end{bmatrix}$

(f.) $c_{\mathcal{B}} = \begin{bmatrix} 8 \\ 0 \\ -2 \end{bmatrix}$

(g.) $c_{\mathcal{N}} = \begin{bmatrix} 0 \\ -20 \\ 12 \\ 20 \\ 0 \end{bmatrix}$

(h.) $x^*_{\mathcal{B}} = \begin{bmatrix} 10 \\ 0 \\ 11 \end{bmatrix}$

(i.) $z^*_{\mathcal{N}} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$.

2. (a.) Auxiliary problem then primal simplex method
Dual-primal II phase simplex method
Parametric self-dual simplex method.

(b.) $$\zeta = -(-2+\mu)x_1 - (1+\mu)x_2 - (-7+\mu)x_3 - (5+\mu)x_4$$

$$\begin{aligned} x_5 &= -7+\mu - x_1 + x_2 \qquad\qquad - x_4 \\ x_6 &= 3+\mu \qquad\quad - x_2 + 2x_3 - 3x_4 \end{aligned}$$

This is optimal for $\mu \geq 7$.

(c.) First look for a variable whose bound on μ is ~~a~~ the lower bound. This will be the entering/leaving variable of the pivot, as appropriate. Then look for the ~~enter~~ leaving/entering variable using the primal/dual ratio test for the specific value of μ at the lower bound.

3. (a.) The initial dictionary is

$$\begin{array}{l} \zeta = \quad 4x_1 - 22x_2 - 4x_3 \\ \hline x_4 = 1 + 2x_1 - 11x_2 - 3x_3 \\ x_5 = 1 - x_1 + 5x_2 \\ x_6 = 2 - 3x_1 + 14x_2 + 2x_3. \end{array}$$

We pivot $x_1 \leftrightarrow x_5$ to reach $\mathcal{B}$, so

$$x_1 = 1 - x_5 + 5x_2.$$

Substituting into the third constraint we have

$$\begin{aligned} x_6 &= 2 - 3(1 - x_5 + 5x_2) + 14x_2 + 2x_3 \\ &= -1 + 3x_5 - x_2 + 2x_3 \end{aligned}$$

so x_6 is infeasible, i.e. $\mathcal{B}$ is no longer optimal.

(b.) The dictionary for $\mathcal{B}$ is

$$\begin{array}{l} \zeta = 4 - 4x_5 - 2x_2 - 4x_3 \\ \hline x_4 = 3 - 2x_5 - x_2 - 3x_3 \\ x_1 = 1 - x_5 + 5x_2 \\ x_6 = -1 + 3x_5 - x_2 + 2x_3. \end{array}$$

Perform a dual pivot with x_6 leaving.

Ratio test: x_5: $x_6 \leq 4/3$

x_2: no restraint

x_3: $x_6 \leq 4/2 = 2$.

So we pivot $x_6 \longleftrightarrow x_5$.

Rearranging,

$$x_5 = \frac{1}{3} + \frac{1}{3}x_6 + \frac{1}{3}x_2 - \frac{2}{3}x_3$$

so $$\zeta = 4 - 4\cdot\left(\frac{1}{3} + \frac{1}{3}x_6 + \frac{1}{3}x_2 - \frac{2}{3}x_3\right) - 2x_2 - 4x_3$$
$$= \frac{8}{3} - \frac{4}{3}x_6 - \frac{10}{3}x_2 - \frac{4}{3}x_3$$

$$x_4 = 3 - 2\cdot\left(\frac{1}{3} + \frac{1}{3}x_6 + \frac{1}{3}x_2 - \frac{2}{3}x_3\right) - x_2 - 3x_3$$
$$= \frac{7}{3} - \frac{2}{3}x_6 - \frac{5}{3}x_2 - \frac{5}{3}x_3$$

$$x_1 = 1 - \left(\frac{1}{3} + \frac{1}{3}x_6 + \frac{1}{3}x_2 - \frac{2}{3}x_3\right) - x_2 - 3x_3$$
$$= \frac{2}{3} - \frac{1}{3}x_6 - \frac{4}{3}x_2 - \frac{7}{3}x_3.$$

This solution is feasible and hence optimal.

The optimal solution is $(x_1, x_2, x_3) = (\frac{2}{3}, 0, 0)$

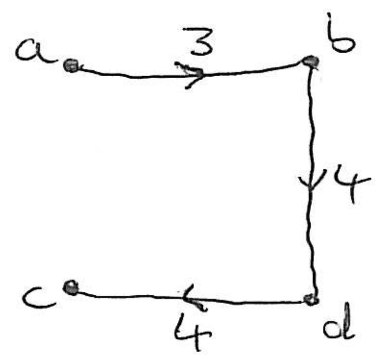
with optimal value $\zeta = 8/3$

4. (a.) minimise $3x_{ab} + 2x_{ad} + x_{ba} + 4x_{bd} + x_{ca} + 2x_{dc}$

subject to
$$
\begin{array}{rrrrrrcr}
-x_{ab} & + x_{ad} & + x_{ba} & & + x_{ca} & & = & -1 \\
x_{ab} & & - x_{ba} & - x_{bd} & & & = & -3 \\
& & & & - x_{ca} & + x_{dc} & = & 4 \\
& x_{ad} & & + x_{bd} & & - x_{dc} & = & 0
\end{array}
$$

$x_{ij} \geqslant 0 \ \forall (i,j).$

(b.)



(c.) Compute the dual variables:

Let d be the root node.

$y_d = 0$

$y_c - y_d = 2 \Rightarrow y_c = 2$

$y_d - y_b = 4 \Rightarrow y_b = -4$

$y_b - y_a = 3 \Rightarrow y_a = -7.$

Then the dual slacks are:

$z_{ba} = y_b + c_{ba} - y_a = 4$

$z_{ca} = y_c + c_{ca} - y_a = 11$

$z_{ad} = y_a + c_{ad} - y_d = -5.$

Since $z_{ad} < 0$, this solution is not dual feasible, and hence not optimal.