LECTURE OUTLINE

The Dot Product

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Math 15

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Today

Polar Coordinates

Projection

Dot Product

Orthonormal Basis

Changing Coordinates

Polar Coordinate Differentiation

Cylindrical Coordinates

We define cylindrical coordinates via

$$(r, \theta, z)_P = (r\cos(\theta), r\sin(\theta), z).$$

We can find a cylindrical coordinate determining (x, y, z) via

$$(x, y, z) = \left(\sqrt{x^2 + y^2}, \arctan\left(\frac{y}{x}\right), z\right)_P,$$

for some $arctan$. Restricting to $z = 0$ we have polar coordinates.

Polar Coordinates: Vectors

When thinking in terms of polar coordinates, we use $\hat{r}$ to describe position

$$\vec{r} = r\hat{r}(\theta) = r(\cos(\theta)\hat{i} + \sin(\theta)\hat{j}),$$

and use $\hat{r}$'s perpendicular companion

$$\hat{\theta} = -\sin(\theta)\hat{i} + \cos(\theta)\hat{j}$$

to describe vectors at $(r, \theta)_P$.

Circular Motion

An object is moving around a circle of radius 1/2 in the x,y-plane (where the units of distance are meters) in a counter clockwise direction at a constant speed of 3 meters per second. Its initial position vector is $(1/2)\hat{i}$.

(a) Describe its position after 6 seconds in polar and Cartesian coordinates.

(b) In both Cartesian and polar coordinates, find a vector representing its velocity when it is located at the point with position vector $(\frac{1}{4})\hat{i} + (\frac{\sqrt{3}}{4})\hat{j}$.

The Angle

Given two unit vectors $\hat{u}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\hat{u}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and letting θ be the angle between them we have

$$\cos(\theta) = 1 - 2\sin(\frac{\theta}{2})^2 =$$

$$1 - 2|\frac{\hat{u}_2 - \hat{u}_1}{2}|^2 = (x_1x_2 + y_1y_2 + z_1z_2)$$

Dot Product

Hence for any $\vec{v} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{w} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, if we let

$$\vec{v} \cdot \vec{w} = x_1x_2 + y_1y_2 + z_1z_2$$

then we have

$$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}$$

where θ is the angle between $\vec{v}$ and $\vec{w}$.

We Used...

Lemma:

$$(c\vec{v}) \cdot \vec{w} = \vec{v} \cdot (c\vec{w}) = c(\vec{v} \cdot \vec{w}).$$