LECTURE OUTLINE

Coordinates and Vectors

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Math 15

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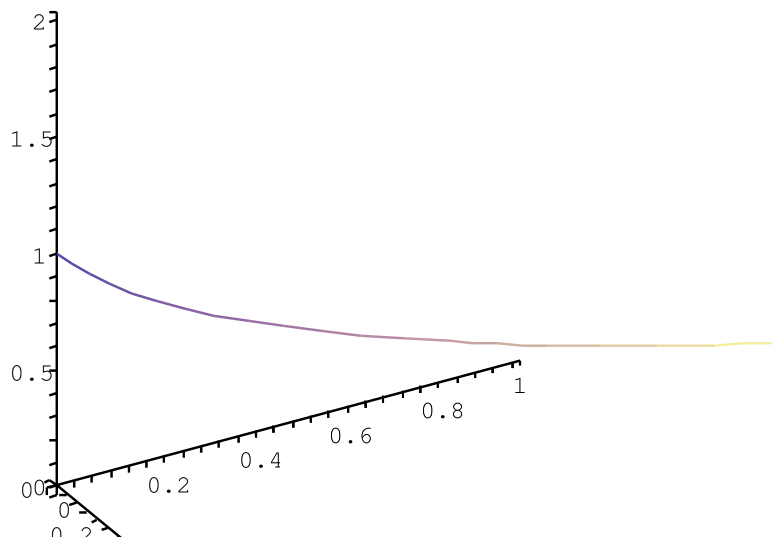
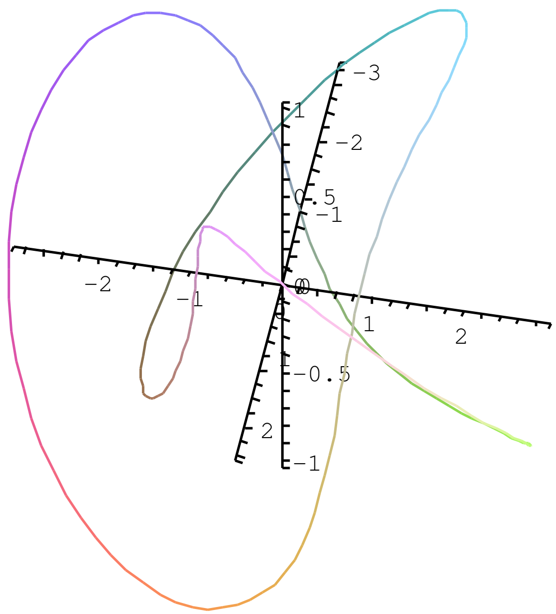
Coordinates

Introducing the (x, y, z) coordinates of three dimensional space.



Movement

Objects can "move" through these coordinates $(x(t), y(t), z(t))$.



Vector

We will also also want to encode direction in magnitude. For example, we will want to say go in direction "blah" for a distance of "blah". We encode such a statement with a *vector*,

$$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$$

Norm

Given a vector $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$ we encoded its *magnitude* (also *norm* or *length*) via

$$|\vec{v}| = \sqrt{x^2 + y^2 + z^2},$$

Scalar Multiplication

To encode the vector's direction, we must first learn *scalar multiplication*:

$$c\vec{v} = cx\hat{i} + cy\hat{j} + cz\hat{k}$$

Notice, the norm satisfies

$$|c\vec{v}| = |c||\vec{v}|.$$

Direction

$\vec{v}$'s direction is given by

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

$\hat{v}$ is called a *unit vector* and has norm 1, and is usually viewed as unitless.

Position Vector

In Euclidean space, once we've chosen a coordinate system we can view the point (x, y, z) as going from the origin in the direction and with the distance determined by the vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$$

We call $\vec{r}$ our points *position vector*.

Displacement

Staring at a point with position vector $\vec{r}$,we may move a point in the direction and for a distance determined by the vector $\vec{d}$. We say we are displacing a point via the *displacement vector* $\vec{d}$.

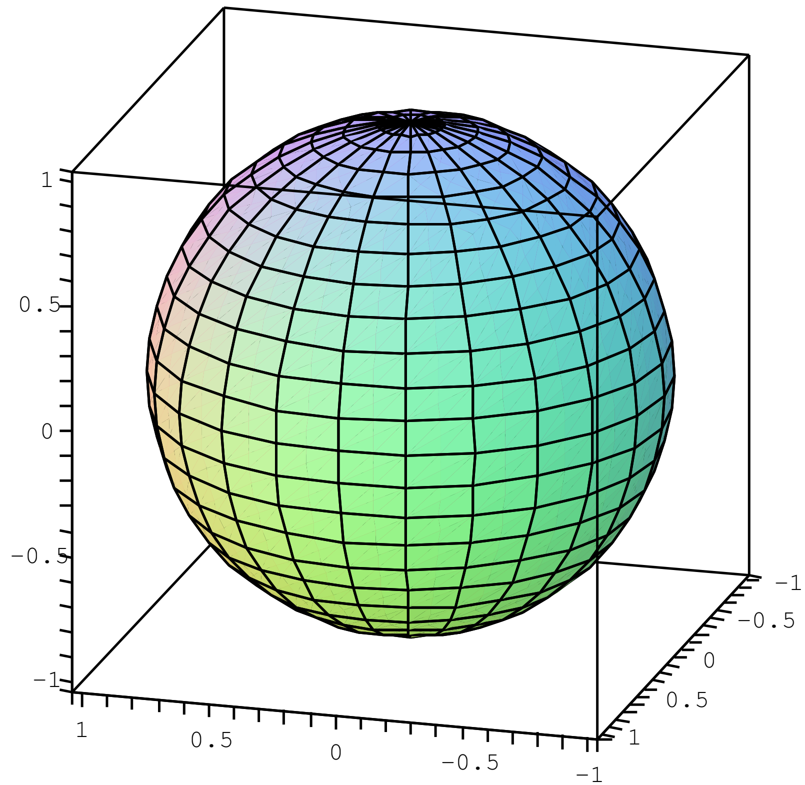
Vector Addition

Wonderfully, in Euclidean space, we can find our destination when staring at $\vec{r}$ and displacing our selves via $\vec{d}$ with *vector addition*. Namely our destination's position vector is

$$\begin{aligned}\vec{r}+\vec{d} &= (x\hat{i}+y\hat{j}+z\hat{k})+(x_d\hat{i}+y_d\hat{j}+z_d\hat{k}) \\ &= (x+x_d)\hat{i}+(y+y_d)\hat{j}+(z+z_d)\hat{k}.\end{aligned}$$

The Sphere, General Relativity

We are really lucky to do so!!!



Properties

Scalar multiplication and vector addition satisfy some rules that can be useful in manipulating them. Let $\vec{t}$, $\vec{v}$ and $\vec{w}$ be vectors and c be a scalar.

$$\vec{v} + \vec{w} = \vec{w} + \vec{v} \qquad commutativity$$

$$\vec{t} + (\vec{v} + \vec{w}) = (\vec{t} + \vec{v}) + \vec{w} \qquad Associativity$$

$$c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w} \qquad Distributivity$$

Linear Motion

An object is moving at a constant speed of 2 meters per second in the direction determined by $\vec{v} = 2\hat{i} + \hat{j} - 3\hat{k}$ and beginning at the point $(17, 2, 3)$ (where the units of distance are meters). Find its location:

(a) After 1 second.

(b) After 7 seconds

(c) After t seconds.