Math 15

106 - 118

106 $\frac{d}{dt}\left(\frac{1}{2}|\vec{v}|^2\right) = \frac{1}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{1}{2}\left(\frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt}\right)$

$$= \frac{1}{2}\left(2\frac{d\vec{v}}{dt}\cdot\vec{v}\right) = \vec{a}\cdot\vec{v}$$

107 a) $\vec{A} = (2,3,4) \quad \vec{B} = (-2,1,8) \Rightarrow |\vec{A}| = \sqrt{29} \quad |\vec{B}| = \sqrt{69}$

$$\vec{A}\cdot\vec{B} = 2\cdot(-2) + 3\cdot 1 + 4\cdot 8 = 31$$

$$|\vec{B}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|} = \frac{31}{\sqrt{29}}$$

$$|\vec{B}|\cos\theta\,\frac{1}{|\vec{A}|}\vec{A} = \left(\frac{31}{\sqrt{29}}\right)\left(\frac{1}{\sqrt{29}}\right)(2,3,4) = \left(\frac{62}{29}, \frac{93}{29}, \frac{124}{29}\right)$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{31}{(\sqrt{29})(\sqrt{69})} = \frac{31}{\sqrt{2001}} \Rightarrow \theta = \cos^{-1}\left(\frac{31}{\sqrt{2001}}\right) = .805$$

b) $\vec{A} = \hat{i} + \hat{j} \quad \vec{B} = \hat{j} + \hat{k} \Rightarrow |\vec{A}| = \sqrt{2} \quad |\vec{B}| = \sqrt{2}$

$$\vec{A}\cdot\vec{B} = 1\cdot 0 + 1\cdot 1 + 0\cdot 1 = 1$$

$$|\vec{B}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|} = \frac{1}{\sqrt{2}}$$

$$|\vec{B}|\cos\theta\,\frac{1}{|\vec{A}|}\vec{A} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\hat{i}+\hat{j}) = \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{1}{(\sqrt{2})(\sqrt{2})} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

c) $\vec{A} = (1,2,1,2) \quad \vec{B} = (-2,3,1,-8) \Rightarrow |\vec{A}| = \sqrt{10} \quad |\vec{B}| = \sqrt{78}$

$$\vec{A}\cdot\vec{B} = 1\cdot(-2) + 2\cdot 3 + 1\cdot 1 + 2\cdot(-8) = -11$$

$$|\vec{B}|\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|} = -\frac{11}{\sqrt{10}}$$

$$|\vec{B}|\cos\theta \frac{1}{|\vec{A}|}\vec{A} = \left(-\frac{11}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right)(1,2,1,2) = \left(-\frac{11}{10}, -\frac{22}{10}, -\frac{11}{10}, -\frac{22}{10}\right)$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{-11}{\sqrt{10}\sqrt{78}} = -\frac{11}{\sqrt{780}} \Rightarrow \theta = \cos^{-1}\left(\frac{-11}{\sqrt{780}}\right) = 1.98$$

108 $\vec{u} = (3,4,5\sqrt{3})$

z-component is 0

Projecting $\vec{u}$ onto the xy-plane gives $\vec{v} = (3,4,0)$

$$|\vec{u}| = \sqrt{9+16+75} = 10 \qquad |\vec{v}| = \sqrt{9+16} = 5$$

$$\vec{u}\cdot\vec{v} = 3\cdot 3 + 4\cdot 4 + (5\sqrt{3})\cdot 0 = 25$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{25}{10\cdot 5} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

109 a) $\vec{d} = (3,3,3) - (2,1,2) = (1,2,1)$

$$W = \vec{F}\cdot\vec{d} = (3,4,1)\cdot(1,2,1) = 3\cdot 1 + 4\cdot 2 + 1\cdot 1 = 12$$

b) $\vec{d}_1 = (1,4,1) - (2,1,2) = (-1,3,-1)$ $\quad \vec{d}_2 = (3,3,3) - (1,4,1) = (2,-1,2)$

$$W = \vec{F}\cdot\vec{d}_1 + \vec{F}\cdot\vec{d}_2 = (3,4,1)\cdot(-1,3,-1) + (3,4,1)\cdot(2,-1,2)$$

$$= 3\cdot(-1) + 4\cdot 3 + 1\cdot(-1) + 3\cdot 2 + 4\cdot(-1) + 1\cdot 2 = 12$$

c) $\vec{d}_1 = (0,0,1) - (2,1,2) = (-2,-1,-1)$ $\quad \vec{d}_2 = (3,3,3) - (0,0,1) = (3,3,2)$

$$W = \vec{F}\cdot\vec{d}_1 + \vec{F}\cdot\vec{d}_2 = (3,4,1)\cdot(-2,-1,-1) + (3,4,1)\cdot(3,3,2)$$

$$= 3\cdot(-2) + 4\cdot(-1) + 1\cdot(-1) + 3\cdot 3 + 4\cdot 3 + 1\cdot 2 = 12$$

110

a) (i) $\vec{v}\cdot\vec{v} = |\vec{v}||\vec{v}|\cos 0 = |\vec{v}|^2$

(ii) $\vec{v}\cdot\vec{w} = |\vec{v}||\vec{w}|\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

b) $\big((x,y,z)-(1,1,1)\big)\cdot\big((x,y,z)-(1,1,1)\big) = 4$

Compare to (i). Let $\vec{v} = (x,y,z) - (1,1,1)$ & $|\vec{v}| = 2$.
This is the set of all points that are 2 units away from the point $(1,1,1)$. In other words, a sphere of radius 2 centered on $(1,1,1)$

$$(x,y,z)\cdot(2,3,1)=0$$

Compare to (ii). This is the set of all (x,y,z) that are perpendicular to the vector $(2,3,1)$. In other words, this set forms the plane through the origin perpendicular to $(2,3,1)$.

III a) $\vec{d} = (2,9,0)-(5,3,10) = (-3,-6,-10)$ $\quad |\vec{d}| = \sqrt{145}$

$$\vec{F}_g = (0,0,-mg) = (0,0,-(3\times10^{-3})(10)) = (0,0,-.03)$$

$$W_g = \vec{F}_g\cdot\vec{d} = (0,0,-.03)\cdot(-3,-6,-10) = .30$$

b) $\vec{F}_g = \vec{F}_{par} + \vec{F}_{perp}$

$$\vec{F}_{par} = \frac{\vec{F}_g\cdot\vec{d}}{|\vec{d}|^2}\vec{d} = \frac{-.3}{145}(-3,-6,-10) = (6.2\times10^{-3}, .012, .021)$$

$$\vec{F}_{perp} = \vec{F}_g - \vec{F}_{par} = (0,0,.03)-(6.2\times10^{-3}, .012, .021)$$

$$= (-6.2\times10^{-3}, -.012, .090)$$

c) $|\vec{F}_f| = \mu_k |\vec{F}_N| = \mu_k |\vec{F}_{perp}| = \mu_k (.091)$

$\Rightarrow \vec{F}_f = -\mu_k (.091)\,\hat{d}$

$W_f = \vec{F}_f \cdot d = -\mu_k (.091)\,\hat{d}\cdot\vec{d} = -\mu_k (.091)\,|\vec{d}|$

$= -\mu_k (.091)(\sqrt{145}) = -1.1\,\mu_k$

$W_{tot} = W_g + W_f = .30 - 1.1\,\mu_k$

$W_{tot} = \Delta KE = \frac{1}{2} m |\vec{v}_f|^2 - \frac{1}{2} m |\vec{v}_i|^2$

$|\vec{v}_i| = 0 \quad |\vec{v}_f| = 7$

$W_{tot} = \frac{1}{2}(3 \times 10^{-3})(7)^2 = .074$

$\Rightarrow .074 = .30 - 1.1\,\mu_k \Rightarrow \mu_k = .21$

d) $\vec{F}_{tot} = \vec{F}_{par} + \vec{F}_f = (-6.2\times10^{-3}, -.012, .021) - \mu_k \frac{.091}{\sqrt{145}}(-3,-6,-10)$

$= (-6.2\times10^{-3}, -.012, .021) - (.21)(7.6\times10^{-3})(-3,-6,-10)$

$= (-1.4\times10^{-3}, -2.4\times10^{-3}, .037)$

$\Delta\vec{p} = \vec{J} = \int_0^t \vec{F}_{tot}\,dt'$. Since $\vec{F}_{tot}$ is constant, $\Delta\vec{p} = \vec{F}_{tot}\int_0^t dt' = \vec{F}_{tot}\Delta t$

$\Rightarrow |\Delta\vec{p}| = |\vec{F}_{tot}|\Delta t \Rightarrow \Delta t = \frac{mv}{|\vec{F}_{tot}|} = \frac{(3.0\times10^{-3})(7)}{.037} = .57\,s$

112 a) $\vec{x}(t) = (\cos t, \sin t) \quad 0 \le t < 2\pi$ parametrizes CCW

$\Rightarrow \vec{x}(t) = (\cos(-t), \sin(-t)) = (\cos t, -\sin t) \quad 0 \le t < 2\pi$

parametrizes CW

b) $W = \int_\gamma \vec{F} \cdot d\vec{x} = \int_0^{2\pi} \vec{F}(x(t), y(t)) \cdot \vec{v}(t)\, dt$

$\vec{F}(x(t), y(t)) = (-y(t), x(t)) = (\sin t, \cos t)$

$\vec{v}(t) = \frac{d\vec{x}}{dt} = (-\sin t, -\cos t)$

$\Rightarrow W = \int_0^{2\pi} (\sin t, \cos t) \cdot (-\sin t, -\cos t)\, dt = \int_0^{2\pi} (-\sin^2 t - \cos^2 t)\, dt$

$= -\int_0^{2\pi} 1\, dt = -2\pi$

c) Yes, in Ex. 45 when the particle moves CCW the force field does work on the particle. To move CW do the same work against the force field.

d) $W_{\substack{2\text{ times}\\\text{around}}} = 2\, W_{\substack{\text{once}\\\text{around}}} = 4\pi$

113 a) $\vec{x}(t) = (t, 0, 0) \quad -1 \le t \le 1$ parametrizes γ

$\vec{v} = (1,0,0)$

$$\int_\gamma \vec{F}\cdot d\vec{x} = \int_{-1}^{1} \vec{F}\cdot\vec{v}\,dt = \int_{-1}^{1} (1,1,1)\cdot(1,0,0)\,dt = \int_{-1}^{1} dt = 2$$

b) $\vec{x}(t) = (\cos t, \sin t, 0) \quad \pi \le t \le 2\pi$ parametrizes γ

$\vec{v} = (-\sin t, \cos t, 0)$

$$\int_\gamma \vec{F}\cdot d\vec{x} = \int_{\pi}^{2\pi} \vec{F}\cdot\vec{v}\,dt = \int_{\pi}^{2\pi} (1,1,1)\cdot(-\sin t, \cos t, 0)\,dt$$

$$= \int_{\pi}^{2\pi} (-\sin t + \cos t)\,dt = \cos t\Big|_{\pi}^{2\pi} + \sin t\Big|_{\pi}^{2\pi} = 2$$

c) $\vec{x}(t) = (t, \sin \pi t, \sin \pi t) \quad -1 \le t \le 1$

$\vec{v} = (1, \pi\cos\pi t, \pi\cos\pi t)$

$$\int_\gamma \vec{F}\cdot d\vec{x} = \int_{-1}^{1} (1,1,1)\cdot(1, \pi\cos\pi t, \pi\cos\pi t)\,dt$$

$$= \int_{-1}^{1} (1 + 2\pi\cos\pi t)\,dt = t\Big|_{-1}^{1} + 2\sin\pi t\Big|_{-1}^{1} = 2$$

d) $\vec{x}(t) = \left(\sin t,\ \cos t,\ \ln\left(\left(\frac{t}{\pi}\right)^2 + \frac{3}{4}\right)\right) \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$

$$\vec{v} = \left(\cos t,\ -\sin t,\ \frac{\frac{2}{\pi}t}{\left(\left(\frac{t}{\pi}\right)^2 + \frac{3}{4}\right)}\right)$$

$$\int_\gamma \vec{F}\cdot d\vec{x} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1,1,1)\cdot\left(\cos t,\ -\sin t,\ \frac{\frac{2}{\pi}t}{\left(\left(\frac{t}{\pi}\right)^2 + \frac{3}{4}\right)}\right) dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cos t - \sin t + \frac{\frac{2}{\pi}t}{\left(\left(\frac{t}{\pi}\right)^2 + \frac{3}{4}\right)}\right) dt$$

$$= \sin t\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \cos t\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \ln\left(\left(\frac{t}{\pi}\right)^2 + \frac{3}{4}\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$$

114 $\vec{x}(t) = (\cos t,\ \sin t,\ t) \qquad 0 \leq t \leq 2\pi$

$$\vec{F}(x(t), y(t), z(t)) = (y(t), z(t), x(t)) = (\sin t,\ t,\ \cos t)$$

$$\vec{v}(t) = (-\sin t,\ \cos t,\ 1)$$

$$\int_\gamma \vec{F}\cdot d\vec{x} = \int_0^{2\pi} (\sin t,\ t,\ \cos t)\cdot(-\sin t,\ \cos t,\ 1)\, dt$$

$$= \int_0^{2\pi} \left(-\sin^2 t + t\cos t + \cos t\right) dt$$

$$= \left(-\frac{1}{2}t + \frac{1}{4}\sin 2t\right)\Big|_0^{2\pi} + \int_0^{2\pi} t\cos t\, dt + \sin t\Big|_0^{2\pi}$$

$$= -\pi + \int_0^{2\pi} t\cos t\, dt$$

Integrate this las term by parts,

Let $u = t$ $dv = \cos t$ $\Rightarrow$ $du = dt$ $v = \sin t$

$$\Rightarrow \int_{\gamma} \vec{F}\cdot d\vec{x} = -\pi + \left(t\sin t \Big|_0^{2\pi} - \int_0^{2\pi} \sin t \, dt \right) = -\pi - (-\cos t)\Big|_0^{2\pi} = -\pi$$

b) $\vec{x} = (y^2, y, 2y)$ Let $t = y$ $\Rightarrow$ $\vec{x}(t) = (t^2, t, 2t)$

$\vec{v} = (2t, 1, 2)$

$\vec{F}(x(t), y(t), z(t)) = (\sin t^2, t, (2t)^3) = (\sin t^2, t, 8t^3)$

$$\int_{\gamma} \vec{F}\cdot d\vec{x} = \int_0^4 (\sin t^2, t, 8t^3)\cdot(2t, 1, 2)\, dt$$

$$= \int_0^4 (2t \sin t^2 dt + t + 16t^3)\, dt = \int_0^4 2t \sin t^2 dt + \frac{1}{2}t^2 \Big|_0^4 + 4t^4 \Big|_0^4$$

$$= \int_0^4 2t \sin t^2 dt + 8 + 1024$$

To evaluate the last integral let $u = t^2$ $du = 2t$

$$\Rightarrow \int_{\gamma} \vec{F}\cdot d\vec{x} = \int_0^2 \sin u \, du + 1032 = -\cos u \Big|_0^2 + 1032$$

$$= -\cos 2 + 1 + 1032 = 1033 - \cos 2$$

c) $\vec{x}(t) = (\cos t, \sin t, 0) \qquad 0 \le t \le 2\pi$

$\vec{v} = (-\sin t, \cos t, 0)$

$\vec{F} = (e^{\cos t}, e^{\sin t}, 1)$

$$\int_\gamma \vec{F}\cdot d\vec{x} = \int_0^{2\pi} (e^{\cos t}, e^{\sin t}, 1)\cdot(-\sin t, \cos t, 0)\,dt$$

$$= \int_0^{2\pi} (-e^{\cos t}\sin t + e^{\sin t}\cos t)\,dt$$

$$= \int_0^{2\pi} -e^{\cos t}\sin t\,dt + \int_0^{2\pi} e^{\sin t}\cos t\,dt$$

For the 1st integral: let $u_1 = \cos t \quad du_1 = -\sin dt$

For the 2nd integral: let $u_2 = \sin t \quad du_2 = \cos t\,dt$

$$\Rightarrow \int_\gamma \vec{F}\cdot d\vec{x} = \int_{t=0}^{2\pi} e^{u_1}\,du_1 + \int_{t=0}^{2\pi} e^{u_2}\,du_2$$

$$= e^{u_1}\Big|_{t=0}^{2\pi} + e^{u_2}\Big|_{t=0}^{2\pi}$$

$$= e^{\cos t}\Big|_0^{2\pi} + e^{\sin t}\Big|_0^{2\pi} = (e^1 - e^1) + (e^0 - e^0)$$

$$= 0$$

d) $\vec{x}(t) = (t, -2t+2) \quad 0 \le t \le 1$ parametrizes γ

$\vec{v}(t) = (1, -2)$

$\vec{F} = (-t, 2t-2)$

$\int_\gamma \vec{F} \cdot d\vec{x} = \int_0^1 (-t - 4t + 4)\, dt = -\frac{5}{2}t^2 + 4t \Big|_0^1 = \frac{3}{2}$

115 $\quad \vec{F}_g = -\frac{mM_g}{r^2}\hat{x}(t) \qquad r = |\vec{x}|$

a) $\vec{x}(t) = (t, -2t+2, 0) \quad 0 \le t \le 1$ parametrizes γ

$\Longrightarrow |\vec{x}| = r = \sqrt{t^2 + (-2t+2)^2} = \sqrt{5t^2 - 8t + 4}$

$\vec{v} = (1, -2, 0)$

$W = \int_0^1 \vec{F} \cdot \vec{v}\, dt = \int_0^1 \frac{-mM_g}{(5t^2-8t+4)} \left(\frac{1}{|\vec{x}|}\vec{x}\right) \cdot \vec{v}\, dt$

$= -mM_g \int_0^1 \frac{1}{(5t^2-8t+4)^{3/2}} (t, -2t+2, 0) \cdot (1, -2, 0)\, dt$

$= -mM_g \int_0^1 \frac{5t-4}{(5t^2-8t+4)^{3/2}}\, dt \qquad$ Let $u = 5t^2 - 8t + 4$

$du = 10t - 8$

$$\Rightarrow W = -mM_g \int_4^1 \frac{1}{2} u^{-3/2}\, du = -\frac{1}{2} mM_g \left(-2u^{-\frac{1}{2}}\right)\Big|_4^1$$

$$= mM_g \left(1 - \frac{1}{\sqrt{4}}\right) = \frac{1}{2} mM_g$$

b) $\vec{x}_1 = (0, 2-t, 0) \quad 0 \le t \le 1$ parametrizes the 1^{st} path

$\vec{x}_2 = (\cos t, -\sin t, 0) \quad 0 \le t \le \frac{\pi}{2}$ parametrizes the 2^{nd} path

$\vec{v}_1 = (0, -1, 0) \qquad \vec{v}_2 = (-\sin t, -\cos t, 0)$

$|\vec{x}_1| = 2-t \qquad |\vec{x}_2| = 1$

Find W_1

$$W_1 = \int_0^1 -\frac{mM_g}{(2-t)^2} \hat{x}_1 \cdot \vec{v}_1 dt = -mM_g \int_0^1 \frac{1}{(2-t)^2} \frac{1}{|\vec{x}|} \vec{x} \cdot \vec{v}\, dt$$

$$= -mM_g \int_0^1 \frac{1}{(2-t)^3} (2-t)\, dt = -mM_g \int_0^1 (2-t)^{-2} dt$$

Let $u_1 = 2-t \quad du = -dt$

$$\Rightarrow W_1 = -mM_g \int_1^2 -u^{-2} du = mM_g \left(u^{-1}\right)_1^2 = \frac{1}{2} mM_g$$

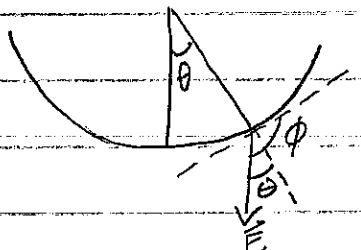
Find W_2

$$\vec{x}_2 \cdot \vec{v}_2 = -\cos t \sin t + \sin t \cos t = 0$$

$$\Rightarrow W_2 = \int_0^{2\pi} -\frac{mM_g}{|\vec{x}|^2} \left(\frac{1}{|\vec{x}|}\right) \vec{x} \cdot \vec{v}\, dt = 0$$

$$\Rightarrow W_{tot} = W_1 + W_2 = \frac{1}{2} mM_g$$

116



$$\phi - \theta = \frac{\pi}{2}$$

$$\Rightarrow \pi - \phi = \pi - \frac{\pi}{2} - \theta = \frac{\pi}{2} - \theta$$

$$\cos\phi = -\cos(\pi - \phi) = -\cos\left(\frac{\pi}{2} - \theta\right) = -\sin\theta$$

117 Let $\vec{P}_1 = (L\cos\theta_0, 0, L - L\sin\theta_0)$ & $\vec{P}_2 = (L\cos\theta, 0, L - L\sin\theta)$

$$W = \int_\gamma \vec{F}\cdot d\vec{r} = \int_\gamma (0, 0, -mg)\cdot(dx, dy, dz)$$

$$= \int_{P_{1x}}^{P_{2x}} 0\,dx + \int_{P_{1y}}^{P_{2y}} 0\,dy + \int_{P_{1z}}^{P_{2z}} (-mg)\,dz$$

$$= \int_{L - L\sin\theta_0}^{L - L\sin\theta} -mg\,dz = -mg\left(z\Big|_{L - L\sin\theta_0}^{L - L\sin\theta}\right)$$

$$= -mg\left(L - L\sin\theta - L + L\sin\theta_0\right) = mgL(\sin\theta - \sin\theta_0)$$

118 a) $\vec{x}(t) = (2\cos t, 2\sin t) \quad 0 \le t \le 2\pi$

$$\vec{v} = \frac{d\vec{x}}{dt} = (-2\sin t, 2\cos t)$$

$$\vec{F}(\vec{x}(t)) = (2\sin t, 2\cos t)$$

$$\int_{t_0}^{t_1} \vec{F}(\vec{x}(t)) \cdot \frac{d\vec{x}}{dt}\, dt = \int_0^{2\pi} (-4\sin^2 t + 4\cos^2 t)\, dt$$

$$\int_{x_0}^{x_1} F_1\, dx + \int_{y_0}^{y_1} F_2\, dy = \int_2^0 y\, dx + \int_0^2 x\, dy$$

but $x^2 + y^2 = 4 \Rightarrow x = \sqrt{4 - y^2}$ & $y = \sqrt{4 - x^2}$

$$\Rightarrow \int_{x_0}^{x_1} F_1\, dx + \int_{y_0}^{y_1} F_2\, dy = \int_2^0 \sqrt{4 - x^2}\, dx + \int_0^2 \sqrt{4 - y^2}\, dy$$

b) $\vec{x}(t) = (2 - 2t, 2t) \quad 0 \le t \le 1$

$$\vec{v} = \frac{d\vec{x}}{dt} = (-2, 2)$$

$$\vec{F}(\vec{x}(t)) = (2t, 2 - 2t)$$

$$\int_{t_0}^{t_1} \vec{F}(\vec{x}(t)) \cdot \frac{d\vec{x}}{dt}\, dt = \int_0^1 (-4t + 4 - 4t)\, dt = \int_0^1 (-8t + 4)\, dt$$

For the line segment, $y = -x + 2 \Rightarrow x = -y + 2$

$$\int_{x_0}^{x_1} F_1\,dx + \int_{y_0}^{y_1} F_2\,dy = \int_2^0 y\,dx + \int_0^2 x\,dy = \int_2^0 (-x+2)\,dx + \int_0^2 (-y+2)\,dy$$